MANUAL

OF

ASTRONOMY.

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SECOND EDITION.

THIRD THOUSAND.

LONDON:

LONGMAN, BROWN, GREEN, LONGMANS, & ROBERTS.

1857.

DUBLIN:
Printed at the University Press,
BY M. H. GILL.

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MANUAL OF ASTRONOMY.

INTRODUCTION.

THE oldest and simplest division of Astronomy is founded on the obvious distinction of the heavenly bodies into the Sun, the Moon, the Earth, and the Stars: the latter being divided into the Fixed Stars, Planets, Comets, and Shooting Stars. If it were our object to write a history of Astronomy, or an account of methods of observation, we should follow the order of discovery, and lay before the reader the successive patient and laborious steps by which mankind have arrived at their present knowledge of Astronomy; following thus the order of time rather than the order of nature,—a method which has been adopted by many writers on Astronomy, but which appears to us badly suited to the purposes of instruction. In order to teach Astronomy, it is necessary to follow the order of nature, and lay first before the learner the exact truth, and then teach him why it is true; rather

than to perplex him at the outset by setting before him problems for solution which have required the united intelligence of the human race for a period of three thousand years to solve. We shall, therefore, commence by describing the Solar System as it is, and afterwards the appearances which have led astronomers to believe that in the Solar System so described is found the true explanation of those appearances.

CHAPTER I.

THE SOLAR SYSTEM.

1. Masses of the Sun and Planets.—2. Kepler's Laws.—3. Division of the principal Planets into Interior and Exterior Planets.—4. Asteroids.—5. Comets.—6. Shooting Stars and Meteoric Stones.

ASTRONOMERS have ascertained that the Sun forms the centre of a group of bodies, including the Earth, Planets, Comets, and Shooting Stars, which revolve round him with different velocities, at different distances, and with different Periodic Times; that the Sun is the centre of Light, Heat, and Attraction, to all these bodies, which thus depend upon his influence; while, on the other hand, the Fixed Stars shine by their own light, like the Sun, and are probably, like him, each the centre of a system of revolving bodies, which resemble the planets and comets of the Solar System, and are dependent exclusively for light and heat on the central Star round which they revolve.

1. **Masses of the Sun and Planets.**—The Sun, and his attendant planets, including the Earth, the Comets, and Shooting Stars or Meteorites, constitute the assemblage of heavenly bodies to which is given the name of the Solar System.

The Sun is worthy of being the centre of the Solar System, as is shown in the following Table, which represents the mass or quantity of matter in the Sun and principal planets, estimated on a scale of which the unit is the quantity of matter contained in the Sun. It is needless to state that this Table is the result of many and long-continued observations of astronomers, and the manner in which it has been calculated cannot be here described.

TABLE I.—*The Masses, Diameters, and Specific Gravities of the Sun and Planets.*

Planets, &c.	Quantity of Matter.	Diameter.	Specific Gravity.
		Miles.	
Sun, . . .	Unity.	882000	0.25
Mercury, . .	$\frac{1}{4863751}$	3140	1.12
Venus, . . .	$\frac{1}{401838}$	7800	0.92
Earth, . . .	$\frac{1}{355551}$	7926	1.00
Mars, . . .	$\frac{1}{2680337}$	4100	0.95
Jupiter, . .	$\frac{1}{1047.87}$	87000	0.24
Saturn, . .	$\frac{1}{3501.6}$	79160	0.14
Uranus, . .	$\frac{1}{22805}$	34500	0.18
Neptune, . .	$\frac{1}{19416}$	41500	0.23

The second column of this Table contains the masses or quantities of matter in the planets, expressed as fractions of the Sun's mass, taken as unity. From this series of figures it appears that the sum of the masses of all the planets added together is only about $\frac{1}{739}$th part of the mass of the Sun, and that the greater part of the mass of the planets is concentrated in a single planet, Jupiter, which contains $\frac{1}{1047}$th part of the mass of the Sun.

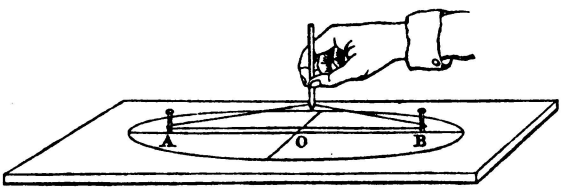
The third column of the Table contains the diameters of the Sun and Planets in English miles; and the fourth column their specific gravity, compared with that of the Earth, considered as unity.

As it has been shown by observation, that the attractions with which the heavenly bodies act upon each other are proportional to their quantities of matter, we see from the foregoing Table the reason why the Sun is the centre of our system, and why all the planets revolve round him as their centre.

The planets named in the preceding Table are arranged in the order of their distance from the Sun; and

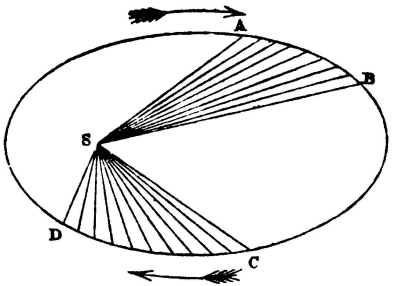
on examining the Table the following curious facts present themselves:—The average specific gravity of the four inner planets is 0.9975, or almost exactly the mean specific gravity of the Earth; while the average specific gravity of the four outer planets is only 0.1975, being less than one-fifth of the specific gravity of the four inner planets; also, the average diameter of the four outer planets is 10.54 times that of the inner planets, and we shall presently see that there are other important differences between the two groups of planets.

2. **Kepler's Laws.**—The paths in which the planets move round the Sun are called their orbits; they are nearly circular, their true figure being that of ellipses. An accurate notion of an ellipse may be formed in the following manner:—

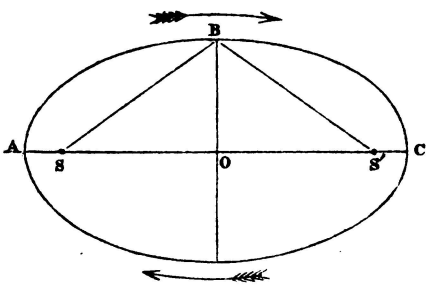


Take two pins A and B, and fasten to them the ends of a string; then with a pencil, keeping the string stretched, describe a curve as in the figure; this curve is an ellipse, and the points A and B are called its foci; if we suppose the Sun placed in one focus A, and the planet to be the point of the pencil moving round the Sun, we shall have an idea of the manner in which the orbit of the planet is described. But at what rate does the planet traverse its path round the Sun? Is its velocity uniform, and, if not, how does it vary? We owe the answer to this question to the celebrated Kepler, who had already discovered that the paths of the planets were ellipses, of which the Sun occupied the focus.

Let ABCD be the elliptic path of a planet, and let it move from A to B in the same time that it moves from



C to D; then the area ASB will be equal to the area CSD. This fact was expressed by Kepler in the statement, that *the planets describe equal areas round the Sun in equal times.*



It is necessary to insert here a few definitions relative to the elliptic motions of the planets.

If AB represent an ellipse, of which S and S' are the foci—

1st. The line OA is called the *semiaxis major* or *mean distance,* of the planet.

2nd. The line OB is called the *semiaxis minor.*

3rd. The ratio of OS to OA is called the *excentricity.*

4th. The time of describing the whole ellipse is called the *Periodic Time.*

5th. The greatest distance SC is called the *Aphelion* distance, and the least distance SA is called the *Perihelion* distance.

When a planet, in its revolution round the Sun, arrives at the point B, or extremity of the minor axis, its distance from the Sun SB is called its mean distance, because it falls short of the greatest distance SC by as much as it exceeds the least distance SA. For, it is evident from the figure that SB = S'B, and from the method of describing the ellipse, it is plain that the length of the chord is equal to SA + S'A, i. e. equal to the axis major, and therefore SB is half the axis major.

Kepler, having proved by observation, that the planets moved in ellipses, having the Sun in one of the foci; and that in traversing these ellipses, they described equal areas in equal times, was led by a comparison of the mean distances of the planets from the Sun, with their Periodic times, to the discovery of a third Law, which may be expressed as follows:—*If the squares of the Periodic Times of the Planets be divided by the cubes of their mean distances from the Sun, the quotients thus obtained are the same for all the planets.*

The following Table shows, in a striking manner, the truth of this statement:—

TABLE II.—*Kepler's Third Law.*

Planets.	Mean Distance.	Periodic Time.	$\frac{T^2}{a^3}$
Mercury, . .	0.38710	87.969	133421
Venus, . . .	0.72333	224.701	133413
Earth, . . .	1.00000	365.256	133408
Mars, . . .	1.52369	686.979	133410
Jupiter, . .	5.20277	4332.585	133294
Saturn, . .	9.53878	10759.220	133401
Uranus, . .	19.18239	30686.821	133422
Neptune, . .	30.03680	60126.710	133405

The second column contains the mean distances of the principal planets from the Sun, the Earth's distance being the unit; the third column contains the number of days in the Periodic Time of each planet; the fourth column contains the squares of the Periodic Times divided by the cubes of the mean distances,—and the near approach to equality of the figures in this column proves the accuracy of Kepler's Law.

The Laws which have been just described were discovered by Kepler from observation, and were afterwards made the foundation of the Theory of Gravitation by Sir Isaac Newton. They may be summed up as follows:—

1st Law.—*The planets move in ellipses, having the Sun in one of the foci.*

2nd Law.—*The planets describe equal areas in equal times.*

3rd Law.—*The squares of the Periodic Times of the planets are proportional to the cubes of their mean distances from the Sun.*

3. **Division of the Principal Planets into the Interior and Exterior Planets.**—The eight planets which have been mentioned in the preceding Tables may be divided into two natural groups:—first, the Interior or Terrestrial Planets; and second, the Exterior Planets. The first group contains Mercury, Venus, the Earth, and Mars; and the second group contains Jupiter, Saturn, Uranus, and Neptune.

They are distinguished from each other by the following differences, some of which have been already mentioned.

1st. The Interior planets, with the exception of the Earth, are not attended by Satellites or Moons; while the Exterior planets all have satellites, and other appendages, such as the Rings which accompany Saturn.

2nd. The density of the Interior planets is much greater than that of the Exterior planets.

3rd. The time of rotation on their axes, or length of day of the Interior planets, is much longer than that of the Exterior planets.

The following Tables show the difference of density and length of day of the two groups of planets:—

TABLE III.—*Interior or Terrestrial Planets.*

Planets.	Length of Day.	Density.
Mercury, . .	$24^h\ 5^m$	1.12
Venus, . . .	23 21	0.92
Earth, . . .	23 56	1.00
Mars, . . .	24 37	0.95
Average, .	$23^h\ 59^m\ 45^s$	0.9975

TABLE IV.—*Exterior Planets.*

Planets.	Length of Day.	Density.
Jupiter, . .	$9^h\ 56^m$	0.24
Saturn, . .	10 29	0.14
Uranus, . .	9 30	0.18
Neptune, . .	. . . .	0.23
Average, .	$9^h\ 58^m\ 20^s$	0.1975

4. **The Asteroids.**—Many attempts were made after the discovery of Kepler's Laws to find some law regulating the relative distances of the Planets from the Sun, particularly as it was thought there was an unusually wide interval between Mars and Jupiter, separating the Interior and Exterior planets. No accurate law respecting the planetary distances has been yet found, but the following curious approximation to a law was published,

although not discovered by the astronomer Bode,* in 1778.

Take the numbers—

0, 3, 6, 12, 24, 48, 96, 192, 384,

each of which (excepting the second) is double of the preceding; and to each of these numbers add 4,—we thus obtain

4, 7, 10, 16, 28, 52, 100, 196, 388.

These numbers nearly represent the distances of the planets from the Sun, as is shown by the following Table :—

TABLE V.—*Bode's Law.*

Planets.	True distance from Sun.	Distance from Sun according to Bode's Law.
Mercury, . .	3.87	4.00
Venus, . .	7.23	7.00
Earth, . .	10.00	10.00
Mars, . . .	15.23	16.00
		28.00
Jupiter, . .	52.03	52.00
Saturn, . .	95.39	100.00
Uranus, . .	191.82	196.00
Neptune, . .	300.37	388.00

These numbers are pretty close, with the exception of those for Neptune, whose true distance is, however, not yet well established.

* Bode acknowledged that he had taken the law of distances from a translation of Bonnet's "Contemplation de la Nature" made by Professor Titius at Wittemberg in 1772. The entire credit of the law belongs to the translator, as there is no mention made of it in Bonnet's work.

As soon as this Law was announced, it was evident that a planet was wanting* between Mars and Jupiter. Instead, however, of a single planet, thirty-three have been already (1 Jan. 1855) discovered between Mars and Jupiter, all of small dimensions. To this group of planets lying between the Interior and Exterior planets, the name of Asteroids has been given. The Table given on the next page shows their distances from the Sun, their periodic times, the names of their discoverers, and the dates of discovery.

Of these planets, the first discovered, Ceres, Pallas, and Juno, appeared to give a confirmation to Bode's Law, as their average distance from the Sun was 27.7; the Law of Bode requiring the distance to be 28.0, and it was believed that the four asteroids first discovered were fragments of the missing planet,—but an inspection of the following Table shows that the distance of the Asteroids from the Sun varies from 22.02 to 31.92, the Earth's distance being 10.00. And as it can be proved from mechanical considerations, that if a planet were broken up into fragments by an explosion, or any other cause, each fragment would, in its new orbit round the Sun, return periodically to the spot where the explosion took place, it is certain that the Asteroids never could have formed a single planet.

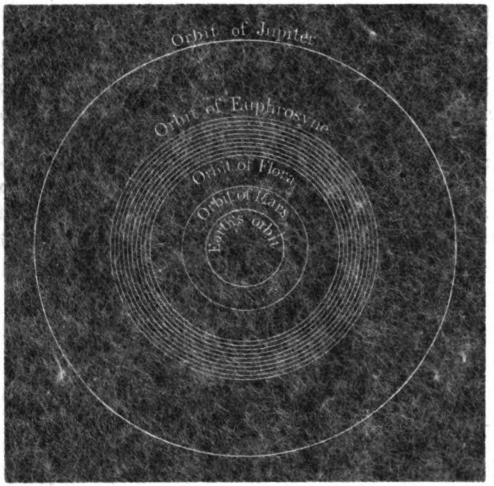
In the figure given at page 13, which is drawn to scale, the belt of Asteroids enclosed between the orbits of Flora and Euphrosyne is represented in its true position and breadth, lying between Mars and Jupiter. It is evident

* It is not, however, to be supposed that the discovery of Ceres was in any way produced by the publication of Bode's law, as the immediate cause of this discovery was an error of the press in Wollaston's "Catalogue of Stars;" but the publication of Bode's law directed attention afresh to the speculation of Kepler that a planet was wanting between Mars and Jupiter. So far was Professor Titius himself from suspecting the truth, that he supposed the vacant space between Mars and Jupiter to be occupied with undiscovered satellites of those two planets.

TABLE VI.—*The Asteroids.*

No.	Planets.	Dist. from Sun.	Periodic Time.	Date.	Discoverer.
1	Flora, . .	22.02	1193.28	18 Oct. 1847.	Hind.
2	Melpomene,	22.96	1270.53	24 June, 1852.	Hind.
3	Victoria, .	23.35	1303.25	13 Sept. 1850.	Hind.
4	Euterpe, .	23.47	1313.74	8 Nov. 1853.	Hind.
5	Urania, .	23.58	1322.83	22 July, 1854.	Hind.
6	Vesta, . .	23.62	1325.67	29 March, 1807.	Olbers.
7	Polymnia,	23.78	1339.90	28 Oct. 1854.	Chacornac.
8	Iris, . . .	23.85	1345.60	13 Aug. 1847.	Hind.
9	Metis, . .	23.87	1346.94	26 April, 1848.	Graham.
10	Phocea, .	23.91	1350.28	6 April, 1853.	Chacornac.
11	Massalia, .	24.08	1365.15	19 Sept. 1852. 20 Sept. 1852.	Gasparis. Chacornac.
12	Hebe, . .	24.25	1379.63	1 July, 1847.	Hencke.
13	Fortuna, .	24.46	1397.19	22 Aug. 1852.	Hind.
14	Parthenope,	24.48	1399.07	11 May, 1850.	Gasparis.
15	Thetis, . .	24.98	1441.86	17 April, 1852.	Luther.
16	Amphitrite,	25.53	1490.54	1 March, 1854.	Marth.
17	Astrea, . .	25.77	1511.37	8 Dec. 1845.	Hencke.
18	Irene, . .	25.82	1515.37	19 May, 1851.	Hind.
19	Egeria, . .	25.82	1515.85	2 Nov. 1850.	Gasparis.
20	Pomona, .	25.85	1518.10	26 Oct. 1854.	Goldschmidt.
21	Lutetia, .	26.12	1542.32	15 Nov. 1852.	Goldschmidt.
22	Thalia, . .	26.26	1554.21	15 Dec. 1852.	Hind.
23	Eunomia,	26.51	1576.49	29 July, 1851.	Gasparis.
24	Proserpine,	26.52	1577.84	5 May, 1853.	Luther.
25	Juno, . .	26.69	1592.73	1 Sept. 1804.	Harding.
26	Ceres, . .	27.67	1681.09	1 Jan. 1801.	Piazzi.
27	Pallas, . .	27.23	1686.09	28 March, 1802.	Olbers.
28	Bellona, .	27.81	1693.69	1 March, 1854.	Luther.
29	Calliope, .	29.12	1814.76	16 Nov. 1852.	Hind.
30	Psyche, .	29.26	1828.45	17 March, 1852.	Gasparis.
31	Hygeia, .	31.51	2043.38	14 April, 1849.	Gasparis.
32	Themis, .	31.60	2052.07	6 April, 1853.	Gasparis.
33	Euphrosyne,	31.92	2083.29	1 Sept. 1854.	Fergusson.

from the figure that the orbits of many of these planets can never intersect; in fact, the distance between the



orbits of Flora and Euphrosyne is nearly the distance of the Earth from the Sun, which is 95000000 miles. There are, without doubt, many more bodies than those mentioned in the Table, circulating round the Sun within the limits of this belt, and the origin of the Asteroids has been attributed by some astronomers to the breaking up of a Solar ring analogous to that of Saturn, which once occupied the space within which the Asteroids, and probably other bodies, now perform their revolutions round the Sun.

5. **Comets.**—The three groups of planets already mentioned are not the only bodies which revolve round the Sun; and astronomers are now agreed to recognise the existence of two other classes of bodies which move

under the influence of the Sun's attraction, viz.: The Comets, and Meteorites or Shooting Stars; but the Planets, properly so called, differ in so many important particulars from these bodies that it is necessary to consider them separately.

In page 6 the excentricity of a planet's orbit is defined to be the ratio which the distance of the centre of its orbit from the Sun bears to the semiaxis major or mean distance. The following Table shows the excentricity of the orbits of the eight principal planets as per centages of the semiaxis major, and also the angles made by the planes of their orbits with the plane of the Earth's orbit, which is called the *Ecliptic*. These angles are called the Inclinations of the orbits to the Ecliptic.

TABLE VII.—*Excentricities and Inclinations.*

Planets.	Excentricity.	Inclination.
Mercury, . .	20.56	7° 0′ 5″
Venus, . .	0.68	3 23 29
Earth, . . .	1.68	0 0 0
Mars, . . .	9.33	1 51 6
Jupiter, . .	4.81	1 18 52
Saturn, . .	5.61	2 29 36
Uranus, . .	4.67	0 46 28
Neptune, . .	0.87	1 46 59

On examining the preceding Table the small value of the excentricities and inclinations is very remarkable; in fact, if we suppose the mean distance or semiaxis major divided into 100 parts, the distance of the Sun from the centre of the orbit of Mercury, which has the greatest excentricity, will be only 20 of these parts, or only one-fifth of the semiaxis major.

The excentricities of the orbits described by comets are much greater than in the case of the planets.

If we suppose the semiaxis major to be divided into 100 parts, the least excentricity which the orbit of a planet or comet can have is zero, in which case the orbit ceases to be an ellipse, and becomes a circle; and the greatest excentricity which an orbit can have, supposing it to be an ellipse, is 100, in which case the orbit just ceases to be an ellipse, and becomes a parabola, or an ellipse having an infinite major axis. If a planet or comet were to move in such a path as this, it would never approach to the Sun a second time. Many of the comets are supposed to move in such orbits, which are not comparable with those of the planets, of which, as we have seen, the orbits are nearly circular; but even the comets which move in elliptic orbits differ widely from the planets, as is shown by the following Table of the four comets whose orbits have been ascertained with the greatest certainty:—

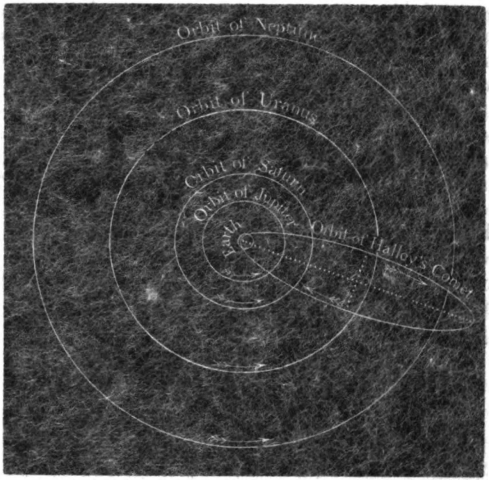
TABLE VIII.—*Elliptic Comets.*

Comets.	Excentricity.	Inclination.	Periodic Time.	Mean distance.
			Years	
Halley, .	96.74	17° 45′ 5″	76.680	17.987
Encke, .	84.77	13 7 24	3.296	2.215
Biela, . .	75.70	12 34 53	6.617	3.524
Faye, . .	55.59	11 22 31	7.441	3.812

The following figure, which is drawn to scale, will serve to convey an idea of the striking differences between the orbits of the planets and of the comets.

It represents the orbits of Halley's Comet, and of the Earth, Jupiter, Saturn, Uranus, and Neptune; the Aphelion distance of the comet lying somewhat beyond the orbit of Neptune, and the Perihelion distance lying somewhat within the Earth's orbit,—its mean distance

and Periodic Time being somewhat less than those of the planet Uranus.



It will be seen from the Table given in the preceding page that the excentricity of the orbit of Faye's comet, which is the least cometary excentricity, exceeds 55 per cent., which is more than double the greatest planetary excentricity, which is found in the orbit of Juno, the excentricity of which is 25.61 per cent.

6. **Shooting Stars and Meteoric Stones.**—Every one is familiar with the phenomenon known by the name of shooting or falling stars, which have been noticed with surprise from the earliest period; but it is only within the last few years that any rational attempt has been made to account for their occurrence. From the numerous observations of late years the following facts have been established respecting these bodies:—

1st. On a clear night, about eight shooting stars per hour may be seen by a careful observer on an average of the entire year.

2nd. At two periods of the year, 9th and 10th August, and 12th to 14th November, the number of shooting stars visible greatly exceeds the average, amounting sometimes to 2600 per hour.

3rd. The shooting stars of 10th August and 13th November, all proceed from nearly the same apparent point among the stars; this point being the star Algol in the constellation of Perseus on 10th August, and the star γ Leonis on 13th November.

4th. The altitudes of the shooting stars above the Earth vary from 16 to 140 English miles.

5th. The shooting stars move at rates varying from 18 to 36 English miles per second.

The third fact just mentioned proves that the shooting stars have their origin outside the Earth, and that they are bodies moving in similar directions in space, which meet and strike against the Earth at the two periods, 10th August and 13th November,—and the fifth fact, showing their great velocities, confirms this supposition, as such high velocities as those found in the shooting stars are only to be found in planetary bodies, as shown by the following velocities of the Interior planets in miles per second:—

Mercury,	30.40	miles per second.
Venus,	22.24	,,
Earth,	18.91	,,
Mars,	15.32	,,

It is at present believed by astronomers that the periodic shooting stars of August and November form portions of belts or zones of small planetary bodies which circulate round the sun like the Asteroids, and whose orbits intersect the path of the Earth at the points in which the Earth is found on 10th August and 13th November.

Among the falling stars are frequently observed bodies

of considerable dimensions, which are commonly called fire-balls, meteors, &c.; they are variously coloured, and leave a track behind them not unlike the tail of a comet in miniature. These bodies are sometimes visible in the daytime, and occasionally have been observed to explode, the explosion being accompanied by the production of a suddenly formed dark cloud, a loud noise, and a shower of meteoric stones. The stones which have been thus observed to fall to the ground are now believed to be shooting stars which have penetrated so far into the Earth's atmosphere that they are unable to return, and are precipitated by the force of gravity upon the Earth's surface.

The planetary bodies have been hitherto considered as known only to us by what may be called their geometrical properties; viz. their size, mass, velocity, and form of their orbits; but in the meteoric stones we have, as it were, hand specimens of the planets presented to us for examination, and the planetary bodies thus become partially brought within the domain of Physics.

The chemical examination of meteoric stones has been prosecuted with zeal and success by many distinguished mineralogists and chemists, especially Berzelius and Rammelsberg, a brief summary of whose results is here given.

1st. The meteoric stones are composed of elements which occur in terrestrial minerals.

2nd. Of the 55 elements admitted by chemists, the following *nineteen* occur in meteoric stones:—

Metalloids.	Metals.	
1. Oxygen.	1. Aluminum.	8. Cobalt.
2. Sulphur.	2. Magnesium.	9. Chrome.
3. Phosphorus.	3. Calcium.	10. Manganese.
4. Carbon.	4. Potassium.	11. Copper.
5. Silicon.	5. Sodium.	12. Tin.
	6. Iron.	13. Titanium.
	7. Nickel.	14. Lead.

3rd. The meteoric stones may be divided into two distinct groups—meteoric iron, and meteoric stones proper.

4th. Meteoric iron is an alloy of iron and nickel, containing about 10 per cent. of nickel, and small quantities of cobalt, manganese, magnesia, tin, copper, and carbon. This alloy has not been found among terrestrial minerals.

5th. The meteoric stones proper are composed of minerals of volcanic origin, and such as are found abundantly in terrestrial lavas and trap rocks, viz.—

Magnetic iron.	Olivine.
Sphene.	Anorthite.
Chrome iron.	Labradorite.
Apatite?	Augite.

Together with a varying proportion of the meteoric iron-nickel alloy.

In the foregoing rapid sketch of the Solar System a classification has been attempted of the bodies belonging to that system, which may be summed up as follows:—

The Earth.
The Sun and Moon.
The Interior Planets.
The Exterior Planets.
The Asteroids.
Comets.
Shooting Stars and Meteoric Stones.

In the succeeding chapters of this Manual we shall describe the most important facts relating to these bodies, giving, as far as possible, the means of understanding the methods by which the facts relating to their dimensions, velocities, orbits, &c., have been ascertained. In the arrangement of the chapters, the convenience of the learner has been chiefly consulted, and we have en-

deavoured to lay the facts of Astronomy before our readers in the manner best suited, in our opinion, to produce an intelligent conviction of their truth.

Whenever it was possible to obtain a simple and direct proof of an astronomical fact, we have always adopted it in preference to the indirect and unsatisfactory induction by which in most cases it became known; and have, as far as was possible, abandoned the Historical and used the Positive method.

CHAPTER II.

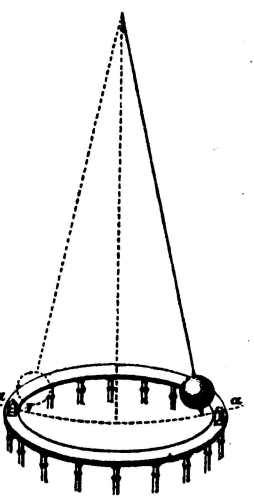
THE EARTH.

1. Rotation of the Earth.—2. Apparent Diurnal Rotation of Celestial Bodies.—3. Annual Motion of the Earth.—4. Appearances produced by the Earth's Annual Motion.—5. Kepler's Laws, 1st and 2nd.—6. Right Ascension and Declination; Latitude and Longitude.—7. Altitude and Azimuth.—8. The Seasons.—9. Change of Celestial Phenomena produced by change of place on the Earth's Surface.—10. Figure of the Earth.—11. Density of the Earth.—12. The Transit Instrument.—13. The Mural Circle.—14. Refraction of the Atmosphere.—15. Parallax.—16. To find the Latitude of a place on the Earth's Surface.—17. To find the Time at a place on the Earth's Surface.—18. To find the Longitude of a place on the Earth's Surface.—19. The Tides.

The Earth is a nearly spherical globe, having a mean diameter of 7912 English miles, and mean specific gravity of 5.440 as compared with water; revolving round an axis once in $23^h\ 56^m\ 4^s$; situated at a mean distance of 95 millions of miles from the Sun, round which it performs a revolution in $365^d\ 6^h\ 9^m\ 10^s.7$, in an ellipse, of which the Sun occupies the focus. It is the third of the planets, reckoned in order of distance from the Sun, and is accompanied by a satellite, the Moon, which revolves round it in an elliptic path, and accompanies the Earth in its revolution round the Sun. The present chapter will be devoted to the proof of the foregoing statements respecting the Earth, and to the consideration of some other subjects, such as the Tides of the Sea, intimately connected with the subject of Astronomy.

1. **Rotation of the Earth on its Axis.**—The simplest proof of the rotation of the Earth is one founded on the following fact respecting pendulums, to which attention was first directed by M. Foucault in 1851. If a heavy body be suspended by a wire from a fixed point

of support, and the pendulum so formed be made to vibrate, it will continue, as is well known, to vibrate in the same plane: if now, instead of being attached to a fixed point, the wire be fastened to a spindle, or similar contrivance, placed vertically, and a slow movement of rotation, say four or five revolutions per minute, round the vertical axis be communicated to the spindle, the plane of vibration of the pendulum will, in this case also, remain fixed, notwithstanding the twisting motion given to the wire by which it is suspended. This remarkable fact is easily established by experiment, and was used by M. Foucault to prove the rotation of the Earth in the following manner:—If we suppose a pendulum composed of a heavy ball and wire suspended to a fixed point, and set vibrating at the pole of the Earth, or point where the axis of rotation intersects the surface, this point of support will be situated on the axis of rotation of the Earth, and will turn with the Earth once in $23^h\ 56^m\ 4^s$. This case is exactly the same as that of the pendulum attached to the turning spindle, and consequently the plane of vibration will remain fixed. The Earth, and the spectator at the pole, will therefore turn round under the plane of vibration in $23^h\ 56^m\ 4^s$. But as the spectator is unconscious of his own motion of rotation, the effect will be, that the plane of vibration will *appear* to revolve in the opposite direction in the same time.



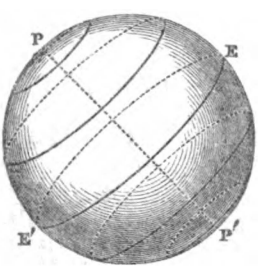
In the accompanying figure the pendulum is shown swinging in the plane *a a*, and under the pendulum is placed a circular table with two little piles of sand at *a* and *a*; the bob of the pendulum is furnished

with a spike, which is not long enough to strike the table, but which reaches the piles of sand. If the pendulum be set in motion near the piles of sand, after a short time its plane of vibration will appear to move from *right to left*, and it will gradually encroach upon the sand, and at length cut it all away. It is to be inferred from this experiment that the table has a movement of rotation from *left to right*.

It was, of course, impossible to try this experiment at the pole of the Earth, but the following considerations show how it was possible to obtain the same result in another way:—

In the annexed figure, PP′ is the axis of rotation of the Earth, and the circle EE′ cutting it at right angles, and passing through the centre of the sphere, is called the *Equator*. The other circles are called *Parallels of Latitude*, because all points situated on any of these circles are at the same distance from the equator; the angular distance from the equator being called Latitude, and measured on the meridian of the place PAP′, shown in the figure, page 26.

If we suppose a pendulum erected at P or P′, its plane of vibration would appear to revolve from right to left



in $23^h\ 56^m\ 4^s$, as already described, in consequence of the observer himself revolving round the fixed plane of vibration; but if a pendulum were erected at the equator EE′ and set vibrating, the spectator, pendulum, and

plane of vibration all move together, and as there is no relative motion of the observer and plane of vibration, the latter will appear fixed, and no motion of the plane will be observed.

If we consider the apparent motion of the plane of vibration per hour, we see that it is zero at the equator, and $\frac{360^\circ}{23^h\,56^m\,4^s} = 15^\circ.0411$ at the poles. At intermediate places it is different, being less as we approach the equator, and greater as we approach the poles. It can be shown by geometrical considerations to be proportional to the sine of the latitude, so that if H denote the hourly motion of the plane of vibration of a pendulum set up at a point on the Earth's surface having a latitude denoted by l, we have

$$H = 15^\circ.0411 \times \sin l. \qquad (1)$$

Experiments have been made at many points on the Earth's surface with a view to determine the apparent motion per hour of the plane of vibration of a pendulum. The following Table contains some of the most carefully determined results :—

TABLE IX.—*Hourly Motion of Pendulum Plane.*

Place.	Latitude.	Observed Motion per Hour.	Calculated Motion per Hour.	Observer.
Ceylon, . . .	6° 56′	1°.870	1°.815	Schaw and Lamprey.
New York, . .	40 44	9 .733	9 .814	Loomis.
Providence, R.I.,	40 49½	9 .955	9 .833	Carswell and Norton.
New Haven, Ct.,	41 18½	9 .970	9 .929	
Geneva, . . .	46 12	10 .522	10 .856	Dufour and Wartman.
Paris, ...	48 50	11 .500	11 .323	Foucault.
Bristol, . . .	51 27	11 .788	11 .763	Bunt.
Dublin, . . .	53 20	11 .915	12 .065	Galbraith & Haughton
Aberdeen, . .	57 9	12 .700	12 .636	Gerard.

The third column of the foregoing Table gives the

observed hourly motion of the pendulum plane, and the fourth column contains the hourly motion calculated from equation (1). The agreement between the columns is remarkably good.

If T' denote the time of the Earth's rotation, the angle described in one hour by the Earth is $\frac{360^\circ}{T}$; and therefore, if the time of rotation be supposed unknown, equation (1) should be written—

$$H = \frac{360^\circ}{T} \times \sin l. \qquad (2)$$

Solving this equation for T, we find

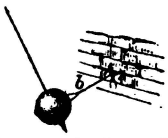
$$T = \frac{360^\circ \times \sin l}{H}. \qquad (3)$$

By means of this latter equation the time of rotation of the Earth is expressed in terms of the latitude of any place, and the hourly motion of the pendulum plane at that place. The following Table has been calculated with the aid of equation (3) from the preceding Table :—

TABLE X.—*Rotation of the Earth deduced from the Pendulum.*

	Time of Rotation.
Columbo, Ceylon, .	$23^h\ 14^m\ 20^s$
New York, . . .	24 8 9
Providence, R. I., .	23 38 29
New Haven, Ct., .	23 50 7
Geneva, ...	24 41 39
Paris, ...	23 33 57
Bristol, ...	23 53 2
Dublin, ...	24 14 7
Aberdeen, ...	23 48 49
MEAN, . . .	$23^h\ 53^m\ 37^s$

The true time of rotation of the Earth is $23^h\ 56^m\ 4^s$, as will be shown in the next section, and the mean rotation deduced from the pendulum experiments differs from this by only $2^m\ 27^s$, which must be considered a very close agreement, considering the difficulty of the experiments. One of the principal difficulties in the experiment consists in starting the pendulum without any lateral motion. This is accomplished by the contrivance shown in the annexed woodcut.



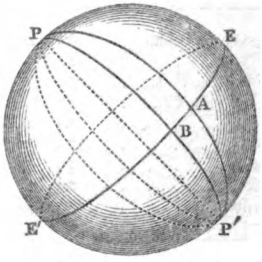
The pendulum is withdrawn from the vertical, and fastened by means of a loop *b* of fine silk to a peg in the wall of the room in which the experiment is tried; after being allowed to rest in this position until all vibratory motion of the wire has ceased, it is started from its position of rest by burning the silk cord.

2. Apparent Diurnal Rotation of Celestial Bodies.

1. The *Latitude* of a place on the Earth's surface is its angular distance from the Equator, measured at the centre of the Earth.

2. A *Parallel of Latitude* is a small circle parallel to the Equator. (*Vid.* fig. p. 23.)

3. The *Meridian* of any place is the circle passing through that place and the axis of rotation of the Earth, and intersects the Equator at right angles.



In the accompanying figure PAP′ and PBP′ are meridians, PP′ being the axis, and EABE′ the equator.

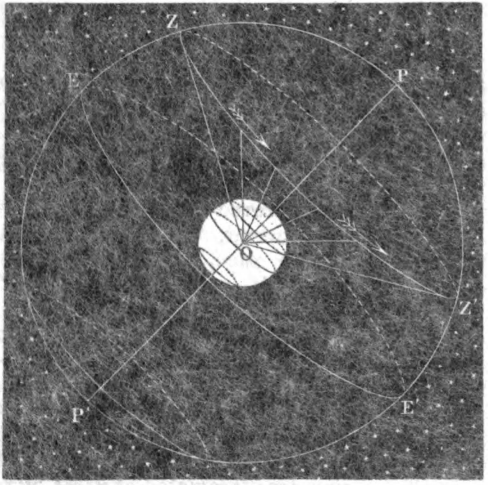
4. The *Horizon* of any place is the tangent plane to the Earth at that place.

5. The plane of the meridian of any place cuts the plane of its horizon in a line called the meridian line; this line is called the North and South line, and a line at right angles to it is called the East and West line.

A spectator situated on the Earth has his view bounded by the horizon, and a hemisphere of sky in which the celestial bodies appear; there is another hemisphere below the horizon, which is not visible. These Hemispheres of light and darkness are shown in page 30.

6. A line drawn perpendicular to the horizon will intersect the visible hemisphere in a point called the *Zenith*.

7. The same line produced below the horizon will intersect the invisible hemisphere in a point called the *Nadir*.



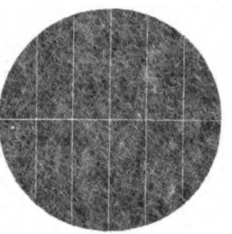
In the annexed figure, the sphere in the centre is the Earth with its equator and parallels of latitude. OZ is

the line drawn from the centre of the Earth to the position of a spectator on a parallel of latitude, and produced to intersect the visible hemisphere in the zenith. As the Earth turns on its axis, the line OZ will turn with it, and the zenith Z of the spectator will travel in the direction of the arrows on a circle ZZ′ among the stars. In like manner the zenith of any other observer will travel on a circle in the sky, and its path may be observed by noting the stars which appear to pass in succession through the zenith. If we suppose these circles, which are all parallel to each other, to be drawn upon the celestial sphere, and the axis of the Earth produced to the points P, P′, and the equator also produced to intersect the sphere in the circle EE′, we shall have drawn on the celestial sphere a copy of the circles on the Earth; the points P, P′ are called the *Poles*, and the circle EE′ is called the *Equinoctial*. The observer stationed on the Earth is unconscious of his own rotation, and will therefore suppose that all the celestial bodies are revolving in circles round the line PP′ in the same time that the Earth turns on her axis, and in the *opposite direction*,—just as a person in a boat or railway train supposes that the objects beside him are moving with a velocity equal to his own, and in the opposite direction.

These facts are easily proved by observation with what is called a transit instrument, which is shown in the figure on the following page, representing the small transit instrument of the Magnetical Observatory of Trinity College, Dublin.

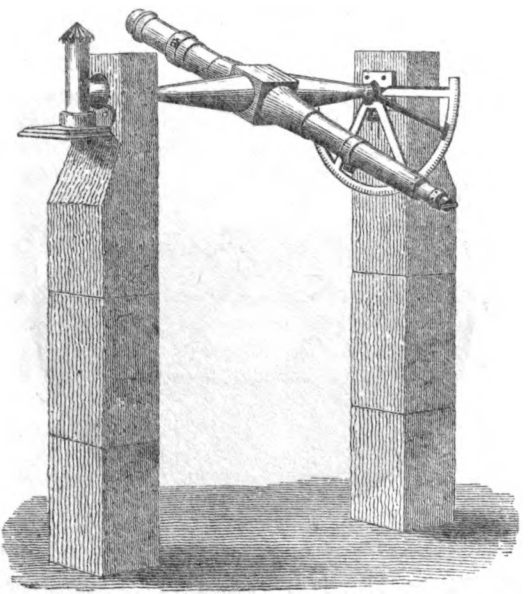
It consists of a telescope, moveable in a vertical plane round a horizontal axis, supported on two pillars,—the vertical plane being made to coincide with the plane of the meridian of the place. At the common focus of the object and eye glasses are placed a number of platinum wires, represented in the figure, one horizontal and five vertical. These wires are illuminated at night by means of a small lamp shown in the sketch, which throws its

light through the hollow axis into the centre of the telescope, where the light is turned by means of a reflecting mirror placed at an angle of 45° into the direction of the telescope, and thus illuminates the wires.



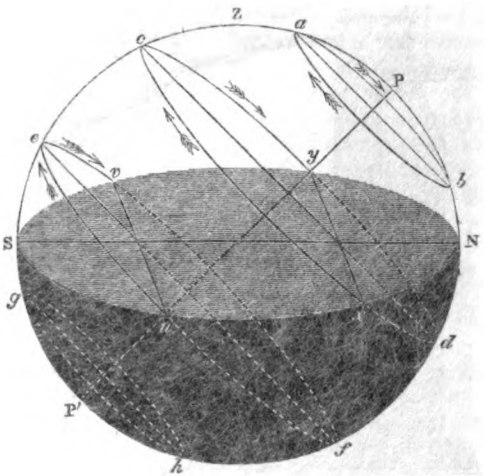
As the observer looks through the instrument, the stars are seen to travel horizontally across the field of view, and the time of passing each vertical wire is carefully noted by the Observatory clock.

If on several succeeding nights the same star be observed, it is found that the interval of time between its



passage of the same wire on the successive nights is always $23^h\ 56^m\ 4^s$, and that this interval is the same for all stars in the sky. The time of rotation of the Earth on its axis found by this method is much more trustworthy than that found by direct observation of the pendulum, but the latter has the advantage of proving directly the fact that the Earth turns on its axis as well as determining its time of rotation.

If an observer could watch the whole apparent path of any star in the sky, he would see it describe a circle round the line PP′ (fig. p. 27); but as only half the celestial sphere is visible, it is evident that part of the path of the star may lie below the horizon, and be invisible. This is shown in the accompanying figure.



Let PP′ be the axis of rotation of the celestial sphere; *NxuSvy* the horizon produced to intersect the sphere,

and dividing it into two hemispheres, NS being the north and south line. If the parallel circles passing through *a*, *c*, *e*, and *g* be the paths of four stars, they will differ in the following manner:—

1st. The star which describes the circle *ab* will never descend below the horizon.

2nd. The star which describes the circle *gh* will never come above the horizon.

3rd. The star which describes the circle *xcyd* will be visible above the horizon, while it moves through *xcy*, and will be invisible while it moves through *ydx*.

4th. The star on the circle *uevf* will be above the horizon through *uev*, and below the horizon through the portion of the circle *vfu*.

These stars are said to *rise* at *x* and *u*, and to set at *y* and *v*. They rise in the eastern part of the horizon, and set in the western, and are said to *south* when they cross the meridian at *c* and *e*.

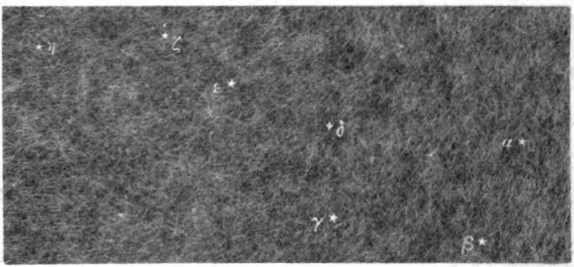
There is a difference in the stars *c* and *e*; in the first star, the visible path *xcy* is greater than the invisible path *ydx*; while in the second star the visible path *uev* is less than the invisible path *vfu*.

It is plain from the figure, that all stars which lie to the north of the equinoctial line will remain above the horizon for a longer period than below it; all stars south of the equinoctial will remain above the horizon for a shorter time than below it; and stars situated on the equinoctial itself, i. e. in the plane of the Earth's equator, will remain above the horizon and below it for an equal time.

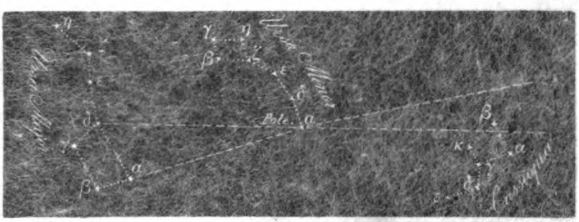
Among the most remarkable of the stars which never set in European latitudes is the group of stars or constellation known as Ursa Major, shown in the figure, page 32.

This constellation is easily recognised, and may be observed occupying different positions in the sky at different hours of the night, and at different periods of the year at the same hour of the night. It may be used to

find the Pole-star, by drawing a line through β and α (called the Pointers) which will pass through the Pole-



star, α Ursæ Minoris, shown in the next figure, which represents Ursa Major, Ursa Minor, and Cassiopea; together with the true position of the pole, or point where the prolongation of the Earth's axis cuts the sky. A line drawn through δ Ursæ Majoris, and α Polaris will nearly pass through the pole, and β Cassiopeæ.



The ancients were well acquainted with the fact that some of the stars rise and set, while others never descend below the horizon. The following, among many allusions to this fact, are interesting:—

Ἄρκτον θ' ἣν καὶ ἅμαξαν ἐπίκλησιν καλέουσιν,
Ἥτ' αὐτοῦ στρέφεται, καί τ' Ὠρίωνα δοκεύει,
Οἴη δ' ἄμμορός ἐστι λοέτρῶν Ὠκεανοῖο.
ILIAD, xviii. 487–9, and ODYS. v. 273-5.

Ἄρκτοι κυανέου πεφυλαγμένοι Ὠκεανοῖο.
ARATUS, PHÆNOM. 48.

Arctos oceani metuentes æquore tingui.
VIRG. GEORG. i. 246.

Immunemque æquoris Arcton.
OVID, METAM. xiii. 293.

Arctos
Æquoris expertes. ID. 726-7.

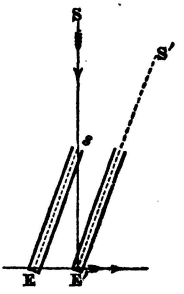
3. **Annual Revolution of the Earth round the Sun.** —The rotation of the Earth on its axis in a period of $23^h\ 56^m\ 4^s$, and the appearances produced in the sky by that rotation, having been shown in the preceding sections of this chapter, we now proceed to the proof of the revolution of the Earth round the Sun.

The *direct* proof of this motion of the Earth in space depends on the fact that light requires a certain interval of time to pass from one point of space to another, combined with the observation of the phenomenon known to astronomers by the term, Aberration of the Fixed Stars.

The first of these facts, viz., that light travels with a certain determinate velocity, has been established beyond doubt by the beautiful experiments of M. Fizeau, made at Suresne and the Buttes de Montmartre, near Paris. By these experiments it has been proved that light travels at the rate of 169944 geographical miles (of 1000 fathoms per mile) in one second of time. It follows, from this fact, that the light from any star reaches the Earth some time after it has left the star; and if the Earth were at rest, this circumstance would not produce any alteration in the apparent position of the star; but if the Earth has a motion of its own in space, this motion, combined with the motion of light, will displace the position of the star in a way which may be illustrated as follows.—

If we suppose a shower of rain to fall in a dead calm, in perpendicular lines, a person at rest would perceive the rain to fall in perpendicular lines; but if he move forward, the rain will beat against his face in the following manner:—

Let us suppose a tube held in the direction Es to move forward with the spectator from Es to E′S′, while the spectator moves from E to E′, and the drop of rain Ss falls from s to E′. If the tube and spectator were at rest, the drop of rain Ss would fall against the inner side of the tube, as shown in the figure; but since the tube moves from E to E′ in the same time that the rain-drop falls from s to E′; it follows that the drop of rain will continue to occupy the centre of the tube, and will ultimately strike against the lower end of the tube in the direction S′E′. The *slant* of the rain is measured by the angle SE′S′, and since the triangle sE′E is right-angled, we have



EE′ = E′s × tangent of slant of rain,

i. e.

Velocity of spectator = velocity of rain × tangent of slant.

If now, S denote a star, and sE′ the distance traversed by the light in the time, during which the Earth moves from E to E′; the angle SE′S′ is called the *Aberration* of the star; and we find, in a similar manner—

Velocity of the Earth = velocity of light × tangent of aberration. (4)

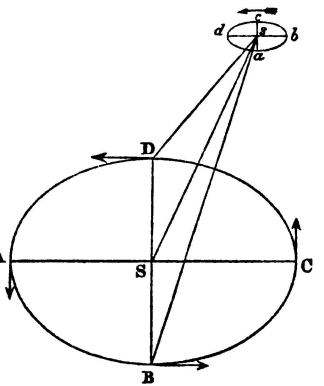
In the figure on next page, let ABCD represent the annual path of the Earth round the Sun, and let s be a star so situated that the line Ss joining the Sun and star shall be perpendicular to the plane of the Earth's orbit, also called the plane of the ecliptic.

The following appearances have been observed by astronomers, and were first noticed by Bradley in the autumn of 1727:—

1st. When the Earth is at the point A, suppose in the beginning of January, moving from A towards B, the star s appears displaced from its true position s, and is observed at a.

2nd. In the beginning of April, when the Earth has arrived at B, the star appears at the point *b*.

3rd. In the beginning of July and October, the star appears in the positions *c* and *d* respectively, and thus appears to traverse the curve *abcd*, in the same direction as the Earth describes her orbit ABCD, and always 90° degrees in advance of the Earth.



The curve *abcd* described by the star *s*, situated in the pole of the ecliptic, appears to be a circle, and has a radius of 20″.4451.

Other stars, not situated in the pole of the ecliptic, appear to describe ellipses more or less elongated, but all having their semiaxes major equal to 20″.4451.

The variations in the form of these ellipses, according to the position of the star, may be found in larger treatises on Astronomy; but the discussion of these variations of form is not suited to the limits of this Manual.

It is proved by the apparent revolution of the star *s* that the Earth revolves in a closed orbit, nearly circular, and with a velocity nearly constant,—the aberratic circle described by the star being a representation of the orbit of the Earth.

The velocity of the Earth in her orbit, and her distance from the Sun, may be found as follows, from equation (4).

The aberration of the star *s*, situated in the pole of the ecliptic, is always 20″.4451, and the tangent of this angle, being small, will be expressed by the fraction whose numerator is 20″.4451, and denominator 206265″, or the number of seconds in an arc equal to the radius;

substituting this value for the tangent of aberration in equation (4), and the number 169944 miles per second for the velocity of light, we obtain—

$$\text{Velocity of Earth} = 169944 \times \frac{20.4451}{206265} = 16.845 \text{ miles per second.}$$

It will be proved in the next section that the Earth performs her revolution round the Sun in 365.256 days, containing each 86400 seconds. The time of describing her orbit will therefore be 365.256×86400 seconds. This time, multiplied by the velocity in geographical miles per second, will give the circumference of the orbit, which is, therefore,

$$365.256 \times 86400 \times 16.845 = 531{,}596504 \text{ g. miles.}$$

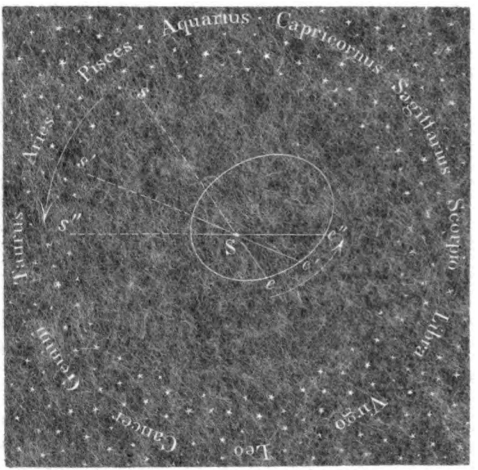
And the radius of the orbit, or Earth's distance from the Sun, will be

$$84{,}606282 \text{ geogr. miles} = 96{,}143502 \text{ statute miles.}$$

The distance of the Earth from the Sun just given depends only on Fizeau's *velocity of light*, on the *aberration* of a star in the pole of the ecliptic, as given by Struve, and on the *time of revolution* of the Earth in her orbit, the method of finding which will be explained in the following section.

4. **Appearances produced by the Earth's Annual Motion.**—If in the figure, page 37, S represent the Sun, and $ee'e''$ the Earth's orbit, a spectator situated on the Earth will see the Sun projected among the fixed stars in the point of the sky opposite to that occupied by the Earth, and as the Earth moves from e to e' and e'', the Sun will appear to move among the stars from s to s' and s'', and in the course of the Earth's revolution will appear to trace out in the sky, by a movement in the same direction as the Earth's motion, a path among the fixed stars, which will coincide with the intersection of

the plane of the Earth's orbit with the sky, and is called the Ecliptic. The path of the Sun in the sky is divided



into 12 Constellations or Signs, containing 30° each, and named:—

TABLE XI.—*Signs of the Zodiac.*

Constellation.	Sign.	Constellation.	Sign.
I. Aries, . . .	♈	VII. Libra, . . .	♎
II. Taurus, . .	♉	VIII. Scorpio, . .	♏
III. Gemini, . .	♊	IX. Sagittarius, .	♐
IV. Cancer, . .	♋	X Capricornus,	♑
V. Leo, ...	♌	XI. Aquarius, .	♒
VI. Virgo, . . .	♍	XII. Pisces, . . .	♓

By watching the Sun's apparent movements in the ecliptic we can ascertain with ease the true motion of the Earth which causes the Sun's apparent motion; and

by numerous observations made on the Sun's annual path among the fixed stars, astronomers have ascertained that from the moment the Sun passes any star in his path until he returns to the same star again, is an interval of $365^d\ 6^h\ 9^m\ 10^s.7$. This is the time of revolution of the Earth round the Sun used in the preceding section.

5. **Truth of Kepler's First and Second Laws in the case of the Earth.**—In the preceding section it has been shown that the Sun appears to travel among the fixed stars with the same angular velocity as the Earth, and always at a point of the ecliptic distant from the position of the Earth by 180°. The daily motion of the Sun among the stars can be measured by methods which will be explained in a subsequent chapter; and if we had any mode of measuring the different distances of the Sun from the Earth at different periods of the year, we should be enabled to test the truth or falsehood of Kepler's first and second laws, as applied to the Earth. The relative distances of the Sun from the Earth are readily ascertained as follows:—With the centre of the Earth as centre, and the Sun's distance as radius, describe a circle; the Sun's apparent diameter will subtend a small angle at the Earth, which angle is easily measured; let this apparent diameter be expressed in seconds, and we shall have the following proportion:—

The distance of the Sun : The diameter of the Sun : : The number of seconds in arc equal radius : Apparent diameter of the Sun in seconds.

If then D denote the distance of the Sun, and d his diameter, we shall have

$$D = \frac{206265 \times d}{\text{diameter in seconds}}. \qquad (5)$$

The following examples will illustrate the use of this formula:—

On the 1st January, 1850, the Sun's apparent diameter was 32′ 34″.6, and on the 1st July, 1850, his diameter was 31′ 30″.2. Find the relative distances of the Sun at these two periods.

Since the numerator in equation (5) is the same for all periods of the year, the distance of the Sun at any period will be inversely proportional to its diameter in seconds; therefore

Distance in January ÷ distance in July = 0.96705.

2. On the 31st March, 1850, and on the 3rd October, 1850, the apparent diameter of the Sun was 32′ 2″.4; find ratio of its distance to the distances in July and January. *Ans.* 0.98352.
1.01703.

By measuring the Sun's apparent diameter from day to day, throughout his entire revolution among the stars, and calculating its distance from the Earth in the manner just illustrated, astronomers have ascertained that the true form of the Earth's orbit is an ellipse, having the Sun in one of the foci. The least and greatest distances nearly correspond with the 1st January and 1st July; and from observations made on the Sun's diameter at these two periods of the year, the excentricity of the Earth's orbit may be thus found—

If a denote the semiaxis major, and b the semiaxis minor, and e the excentricity of the Earth's orbit; we have, by the figure in page 6—

The Earth's aphelion distance $= a(1 + e)$,
The Earth's perihelion distance $= a(1 - e)$,

and if A and P denote the aphelion and perihelion distances, we find

$$\frac{A}{P} = \frac{1 + e}{1 - e}, \tag{6}$$

or, solving this equation for e, we have

$$e = \frac{A - P}{A + P} = \frac{1 - \frac{P}{A}}{1 + \frac{P}{A}}. \tag{7}$$

But $\frac{P}{A}$ is equal to $\frac{31'\,30''.2}{32'\,34''.6} = 0.96705$,

and therefore the excentricity e is equal to 0.01675. This result agrees with that given in Table VII.

It remains to be shown that Kepler's second law of equable description of areas is true of the Earth's motion. This law is readily verified when the Earth is in aphelion and perihelion, at which periods her motion is perpendicular to the line joining the Earth and Sun, and the area described by the Earth's radius vector in one day is the area of an isosceles triangle, whose base is the diurnal path and vertex at the Sun; if a be the distance from the Sun, and θ the angle at the Sun, this area is $\frac{1}{2} a^2 \sin \theta$, and since θ, the Sun's diurnal motion in the ecliptic, is small, we may write—

$$\text{Area} = \tfrac{1}{2} a^2 \theta. \qquad (8)$$

This area ought, if Kepler's second law be true, to be the same on the 1st January and 1st July,—but we have proved the Sun's distance a, to be inversely proportional to his apparent diameter, and therefore the ratio of the Sun's daily motion θ, to the square of his apparent diameter should be the same on the 1st January and 1st July. This is proved as follows:—

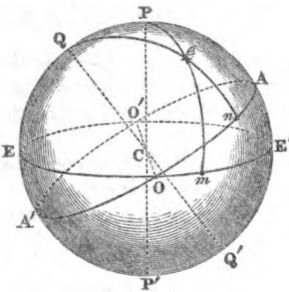
From noon of 1st January to noon of 2nd January, 1850, the Sun moved through 1° 1′ 8″.7; and his apparent diameter at the same period was 32′ 34″.6. The former of these quantities divided by the square of the latter gives for quotient 0.0009603.

From noon of 1st July to noon of 2nd July, 1850, the Sun moved through 57′ 11″.9 of the ecliptic, and at the same period his apparent diameter was 31′ 30″.2; the former of which divided by the square of the latter gives for quotient 0.0009605.

These two results coincide to the sixth place of decimals, and the observations made at other periods of the year, although not so easily reduced, give results equally in accordance with Kepler's second law. Both laws are to be considered as proved with respect to the Earth.

6. **Right Ascension and Declination; Latitude and Longitude.**—The plane of the Earth's equator, which is perpendicular to her axis of rotation, and the plane of

the ecliptic, which is the plane of her orbit round the Sun, are two of the most important planes considered in Astronomy. The position of all celestial objects is determined with reference to one or other of these two planes, as follows :—



Let the accompanying figure represent the celestial sphere; let PP′ be the axis of rotation of the Earth produced to intersect the sphere, and let EE′ denote the equinoctial, or circle in which the Earth's equator cuts the sphere. The points P and P′ are 90° distant from all points of the equinoctial line EE′; and if *e* denote a star or celestial object, the arc P*em*, drawn from P through *e*, will contain 90° from P to *m*, and will also intersect the equinoctial at right angles.

This arc of a great circle P*em* is called a secondary to the equinoctial.

In like manner, let AOA′ be the ecliptic, intersecting the equinoctial in the points O and O′, and let QQ′ be the poles of the ecliptic, 90° distant from each point of that circle. The arc of a great circle Q*en* is called a secondary to the ecliptic, and cuts it at right angles. From this figure the meaning of the following definitions will be readily perceived :—

1st. The *Declination* of an object *e* is the arc *em*, which measures its distance from the equinoctial on a secondary perpendicular to that circle.

2nd. The *Right Ascension* of an object *e* is the arc O*m*, measured on the equinoctial from the foot of the perpendicular *em* to the point O, which is the intersection of the equinoctial and ecliptic.

3rd. The *Latitude* of an object *e* is the arc *en*, which measures its distance from the ecliptic on a secondary perpendicular to that circle.

4th. The *Longitude* of an object *e* is the arc O*n*, measured on the ecliptic from the foot of the perpendicular *en* to the point O, which is the intersection of the equinoctial and ecliptic.

5th. The arc *Pe* is called the *Polar Distance* of *e*, and is the complement of the declination.

From the preceding definitions it appears that right ascension and declination correspond with terrestrial longitude and latitude.

If we were given the right ascension and declination O*m* and *em* of an object *e*, we could find by spherical trigonometry the latitude and longitude *en* and O*n*, provided we knew the angle AOE′, which is called the inclination of the equator to the ecliptic. This angle is 23° 28′.

In astronomical observatories the right ascension and declination are measured by observation, and the latitude and longitude inferred from these by calculation, if necessary. In sections 12 and 13 will be found the methods by which the right ascension and declination are measured.

7. **Altitude and Azimuth.**—The method of fixing the position of a celestial object, with respect to a fixed plane, which has been just described with reference to the ecliptic and equator, is also used for the purpose of referring the object to the plane of the horizon.

In the figure in page 30, let *e* be a celestial object, Z the zenith or pole of the horizon, N*xu*S*vy* the horizon; the arc *e*S is called the *altitude* of the object *e*, and the arc N*xu*S, measured from the north point of the horizon to the foot of the perpendicular *e*S, is called the *azimuth*

of the object *e*. In the instance just given the azimuth is 180°, because the object *e* selected for illustration is situated on the meridian PZS,—and in all cases—

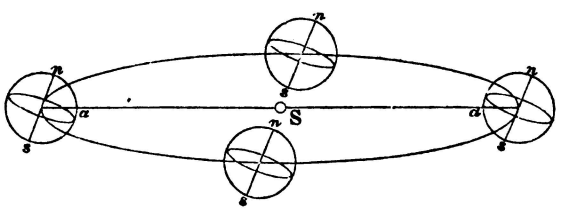
The *Altitude* of an object is its distance from the horizon, measured on a secondary perpendicular to the horizon; and—

The *Azimuth* of an object is the angle N*x*, N*u*, &c., or angular distance of the foot of this perpendicular from the north point of the horizon.

The arc *Ze* is called the *Zenith Distance*, and is the complement of the altitude, because the arc ZS is 90°.

The secondaries to the horizon are called *Vertical Circles*, and that which passed through the east and west points of the horizon, is called the *Prime Vertical*.

8. **The Seasons.**—The plane of the Earth's orbit round the Sun, and the plane of her rotation on her axis, do not coincide, but make with each other an angle of 23° 28′, as will be proved in the Fourth Chapter. The variety of seasons experienced during the course of the year is a consequence of this fact, as is shown in the accompanying figure, in which S represents the Sun,



and the four spheres, the Earth at midsummer, midwinter, the spring, and autumnal equinoxes,—the line *aa* joining the midsummer and midwinter positions,—and *n*, *s* denoting the north and south poles of the Earth.

The line *ns* is inclined to the plane of the Earth's orbit at an angle of 66° 32′, and at midsummer the north pole leans towards the Sun; in consequence of this inclination the line joining the centre of the Earth with

the Sun will intersect the Earth in a point *a*, 23° 28′ north of the equator; and on midsummer day the Sun will appear to pass through the zenith of all places having a latitude of 23° 28′ north. At midwinter, on the contrary, the north pole leans away from the Sun, and therefore the line joining the centres of the Earth and Sun will intersect the Earth in a point *a*, 23° 28′ south of the equator; and all spectators situated on the parallel of latitude 23° 28′ south will see the Sun pass through their zenith on midwinter day.

The parallel of latitude 23° 28′ north is called the tropic of Cancer, and the parallel 23° 28′ south is called the tropic of Capricorn, because the Sun enters those signs of the zodiac on midsummer and midwinter days respectively, and attains his greatest declination north and south of the equator.

A line drawn through S, at right angles to the line *aa*, will coincide with the intersection of the equator with the ecliptic; and when the Earth arrives at either extremity of this line the Sun will be in the equator, at one of its intersections with the ecliptic, and will therefore remain above and below the horizon of all points of the earth an equal time. For this reason the periods at which the Earth in her orbit passes the intersection of the equator and ecliptic, are called the spring and autumnal equinoxes,—the spring equinox occurring when the Sun enters Aries, and appears to pass from the south to the north of the equator, and the autumnal equinox occurring when the Sun enters Libra, and passes from the north to the south of the equator.

The periods at which the Earth passed the points just described were, in 1850—

Spring equinox at	. .	11h	3m	P.M. of 20th March.
Midsummer at	. . .	8	0	P.M. of 21st June.
Autumnal equinox at		10	0	A.M. of 23rd September.
Midwinter at	. . .	3	38	A.M. of 22nd December.

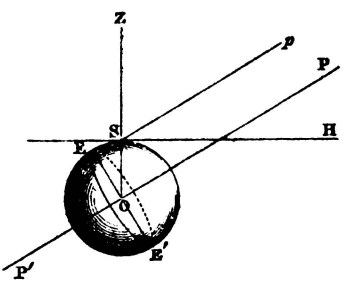
9. **Change of Celestial Phenomena produced by Change of Place on the Earth's Surface.**—If an observer were to travel north or south on the surface of

the Earth, the position of the axis of rotation of the Earth would appear to change its place in the sky in the following manner:—

At one of the poles of the Earth, the axis of the Earth's rotation would be vertical, and pass through his zenith, and consequently all the celestial objects would appear to travel in horizontal circles, parallel to his horizon, traversing these horizontal circles once in $23^h\ 56^m\ 4^s$.

If the observer were situated at the Earth's equator, the axis of the Earth's rotation would lie in the horizontal plane in the north and south line, and all the celestial objects would appear to rise perpendicularly on the eastern side of the horizon, and set perpendicularly on the western side; being equal times above and below the horizon.

But if the observer were situated at any intermediate place between the pole and the equator, such as Dublin or London, the axis of the Earth's rotation would intersect the sky in a point whose *Altitude would be equal to the Latitude of the place*, a fact which may be proved as follows:—



Let PP′ be the axis of rotation, passing through the centre of the Earth at O; let EE′ be the equator, and S the position of any spectator. Through the point S draw the tangent plane SH to represent the spectator's hori-

zon; the line OSZ will pass through the zenith Z; and let Sp be drawn parallel to PP′.

The pole will appear to the spectator at S in the direction Sp, and the angle pSH will be the Altitude of the pole, and the angle SOE will be the Latitude of the spectator at S: it is easy to prove that these two angles are equal to each other, as follows:—

The altitude of the pole pSH is the complement of the zenith distance of the pole pSZ, but pSZ is equal to POZ (because pS is parallel to PO), and POZ is the co-latitude or complement of latitude SOE; and therefore pSH = SOE. Hence we derive the following theorems:

1st. *The Altitude of the Pole seen from any place on the Earth's surface is equal to the Latitude of that place.*

2nd. *The Zenith Distance of the Pole seen from any place on the Earth's surface is equal to the Co-latitude of that place.*

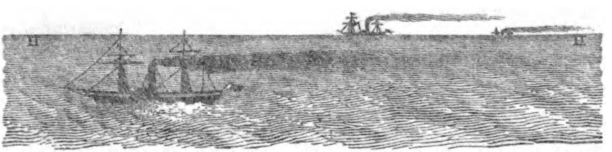
By means of these important propositions the latitude of a place on the Earth's surface may be found by finding the apparent altitude of the pole as seen at the place. This method of finding the latitude will be described in a subsequent section.

By comparing the propositions just given with the figure on page 30, we see that all stars whose north polar distance is less than the latitude will never set; and that all stars whose south polar distance is less than the latitude will never rise. Allusion has been made, page 32, to the observation of the ancients, that the Great and Little Bear do not set; and in interpreting their language by the aid of modern astronomy it will be seen that there was more meaning in the expressions *πεφυλαγμένοι Ὠκεανοῖο*, and '*æquoris expertes*,' as used by a Greek or Italian, than if used by us, who live in a more northern latitude.

In fact, the north polar distance of η Ursæ Majoris, which is the most distant of the stars of these constellations from the pole, is 39° 56′ 48″; and since the lati-

tude of Athens is 37° 58′, and of Naples 40° 50′, an inhabitant of the former city would see this star descend below the northern horizon for a small portion of its course, whereas, with us it never descends below the altitude of 13° 23′; and an inhabitant of Naples would see the star η Ursæ Majoris descend within 3′ of the horizon, so as just to move along its northern edge.

10. **Figure of the Earth.**—A general idea of the globular figure of the Earth may be formed from the fact, observed at every sea coast, that vessels in the distance only show their topmasts, and that as they approach they exhibit more and more of their rigging, and at length show their hulls. These facts are well exhibited in the accompanying figure; and have given rise



to many terms used among sailors and fishermen, such as a vessel being "hull down," &c. Although the spherical or globular form of the Earth might be inferred from such facts, it is from the propositions of the preceding section that the exact figure of the Earth is found, and its dimensions numerically ascertained. Let us suppose an observer, provided with the means of measuring the altitude of the pole, to start from any point on the Earth's surface, and travel north on the meridian, until the pole had increased its altitude by one degree,—from the first proposition of page 46 it follows that the difference of latitude of the two places is one degree: and if, in addition to measuring the altitude of the pole, this observer should chain the distance travelled by him, he would have the means of ascertaining the exact figure of the Earth. Such observations are called geodetical, and have been made in many different parts of the world, and

always with the result, that in every part of the Earth the distance travelled north and south, in order to increase the altitude of the pole by one degree, is nearly the same, from which it follows that the Earth is nearly spherical.

It has been found from these measurements that the mean length of a degree of the meridian is 69.04866 miles, from which the diameter of the Earth may be calculated as follows:—

The diameter of the Earth in miles : The length of a degree in miles : : The number of seconds in arc equal to the diameter : The number of seconds in one degree, or—

Diam. of Earth in miles : 69.04866 miles : : 2 × 206265″ : 3600″.

Hence we find

Mean diameter of the Earth = 7912.4 miles.

The length of a degree of the meridian is not strictly the same at all latitudes, but increases slightly with the latitude, being least at the equator, and greatest at the poles. This arises from the fact that the Earth is bulged out at the equator in consequence of the centrifugal forces developed by her rotation, as explained in the Manual of Mechanics. From these slight variations in the length of the degree, astronomers have inferred, by methods which cannot be here explained, the exact difference between the polar and equatorial diameters of the Earth, and have found the following results:—

Equatorial diameter	= 7925.6 statute miles.
Polar diameter	= 7899.2
Difference	= 26.4
Ratio of Equatorial and Polar diameters	= 300.7 : 299.7

The following Table shows the actual lengths of degrees of the meridian measured in different countries:—

TABLE XII.—*Length of Degree of Latitude.*

Country.	Latitude.	Length of Degree.
Sweden,	66° 20′ 10″ N.	365744 ft.
Sweden,	66 19 37	367086
Russia,	58 17 37	365368
Russia,	56 3 55.5	365291
Prussia,	54 58 26	365420
Denmark,	54 8 13.7	365087
Hanover,	52 32 16.6	365300
England,	52 35 45	364971
England,	52 2 19.4	364951
France,	46 52 2	364872
France,	44 51 2.5	364572
Rome,	42 59 0	364262
America,	39 12 0	363786
India,	16 8 21.5	363044
India,	12 32 20.8	362956
Peru,	1 31 0.4 S.	363626
Cape of Good Hope,	33 18 30	364713
Cape of Good Hope,	35 43 20	364060

In this Table there are some exceptions to the rule that the meridian degree increases with the latitude; and these exceptions are not errors of observation, but indicate peculiarities in the figure of the Earth at the places where they occur.

It has been remarked as an easy mode of remembering an important number, that the length of the degree at a latitude intermediate between London and Dublin is exactly 365000 feet, being as many thousand English feet as there are days in the year.

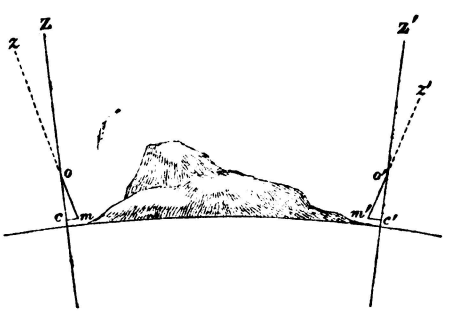
The nautical mile deduced from this value is 6083 ft. 4 in.

11. **Density of the Earth.**—The density or specific gravity of the Earth has been found by a variety of methods. The earliest and simplest of these methods is

that used by Maskelyne, Hutton, and Playfair, in the years 1774, 1775, and 1776.

A mountain called Schiehallien, in Perthshire, was selected for the purpose, partly on account of its regular shape, and partly because it was composed in nearly equal parts of granular quartz, mica schist, and crystalline limestone, the specific gravities of which were easily ascertained, and pretty constant. The cubical contents of this mountain were carefully ascertained by Mr. Hutton, an operation which required an accurate survey of the mountain, and laborious calculations; and from the cubical contents of the mountain and its shape, it was possible to calculate its attraction *cm* and *c'm'* upon plumb lines *oc* and *o'c'* placed at opposite sides.

The result of this laborious calculation was given by Mr. Hutton in the following manner:—If we suppose the Earth to be composed of the same materials as the mountain, and that *oe* or *o'e'* represent the attraction of the Earth upon the plumb line, i. e. its weight; then the Earth's attraction will be to the sum of the opposite attractions of the mountain in the proportion of the figures 87522720 to 8811$\frac{2}{3}$, or as 9933 to 1 very nearly.



If, therefore, *E* denote the attraction of the Earth upon the hypothesis that it has the same density as

Schiehallien; and M denote the sum of the opposite attractions mc and $m'c'$ of the mountain, we shall have

$$E = 9933 \times M. \quad \text{(Hutton's equation.)}$$

Two astronomical observatories were then placed upon the north and south sides of Schiehallien, the observations in which were made by the Rev. Nevil Maskelyne, Astronomer Royal, upon the following principle:—

If the mountain Schiehallien exerted no influence whatever upon the plumb lines co and $c'o'$, they would each point to the true zenith of the place of observation Z and Z'; instead of pointing to Z and Z', they were observed to indicate two points z and z' further away from the mountain, and by measuring the angles Zoz and $Z'o'z'$, the ratio of the true attraction of the Earth to the attraction of Schiehallien may be found as follows:—In the triangles moc and $m'o'c'$ the lines mc, $m'c'$ represent the attraction of Schiehallien, and $oc = o'c'$ the attraction of the Earth. Hence we have

$$mc = co \times \tan moc.$$
$$m'c' = c'o' \times \tan m'o'c'.$$

The sum of the angles moc and $m'o'c'$ was found to be 11".6; and since the angles are small, we may substitute for each of the tangents the value of the angle in seconds divided by the number of seconds in an arc equal to radius. We thence obtain the proportion, that the attraction of the Earth is to the sum of the attractions of Schiehallien, as

$$206265'' : 11''.6, \text{ or } 17781 : 1.$$

If, therefore, the Earth be n times as dense as the mountain Schiehallien, we find

$$nE = 17781 \times M. \quad \text{(Maskelyne's equation.)}$$

Combining this result with that found by Hutton, we obtain for n, which expresses the density of the Earth referred to Schiehallien as unity—

$$n = 1.79009.$$

It now remained to obtain the specific gravity of Schiehallien, which was accomplished by Mr. Playfair, who found the rocks composing the mountain to vary from 2.4 to 3.0 and 3.2. Mr. Playfair considered the mean density of the entire mountain to lie between the limits 2.7 and 2.8, most probably close to the latter; assuming 2.8 as the specific gravity of Schiehallien, we find for the specific gravity of the Earth

$$2.8 \times n = 5.012.$$

The value just found for the density of the Earth should undergo some small corrections for the diminution of gravity caused by the rotation of the Earth, but it is not worth while to introduce them, since they are much smaller than the probable errors caused by an inaccurate estimate of the density of Schiehallien. Nearly all the methods proposed for determining the density of the Earth are open to this objection, that they require an accurate knowledge of the mean density of large masses of rocks.

But there is one method, invented by Cavendish, which cannot be here described, by which the attraction of the Earth is compared with the attraction of a sphere of lead of known size and density; in this case we know accurately the density of the object compared with the Earth; but, on the other hand, the attraction of the sphere of lead is very small compared with that of a mountain such as Schiehallien or Mont Cenis, used in Carlini and Biot's experiments.

The following values of the mean density of the Earth are those found by the best observers:—

TABLE XIII.

Density of Earth.	Observer.	Method employed.
5.012	Maskelyne, Hutton, and Playfair.	Deviation of Plumb Line caused by Schiehallien.
4.837	Carlini and Biot.	Pendulum experiments at Bordeaux and Mont Cenis.
5.480	Cavendish.	Torsion Balance and Pendulum.
5.440	Reich.	" "
5.670	Baily.	" "
5.480	Airy.	Pendulum experiments in Harton Coalpit.

From these various results, we find that the mean density of the Earth is not very different from 5.500 times the density of water.

12. **The Transit Instrument.**—Of the various instruments used in astronomical observatories we shall select for description the Transit Instrument and the Mural Circle, which are indispensable, and by the aid of which all the usual work of an observatory can be accomplished.

The transit instrument is figured in page 29, and a description of its use for the purpose of ascertaining the time of rotation of the Earth on its axis is there given. It is used for ascertaining the right ascensions of celestial objects in the following manner:—The observatory is provided with a clock, marking 24 hours in $23^h\ 56^m\ 4^s$, and reading $0^h\ 0^m\ 0^s$ at the instant that the first point of Aries crosses the meridian. The time registered by this clock is called sidereal time, and the dial of the clock is marked from 0 to 23 hours.

In order to find the right ascension of any celestial object, it is only necessary to watch its meridian passage of the central wire of the transit instrument, and note the exact time of passage, which is accomplished partly

by the eye, and partly by the ear; the eye noting the passage of the wire, and the ear noting the beats of the observatory clock. A practised observer can readily note the time of passage to within one-tenth of a second of sidereal time. The time of meridian passage being thus found, it is converted into right ascension in the proportion of 24 hours to 360°, or one hour equal to 15° right ascension.

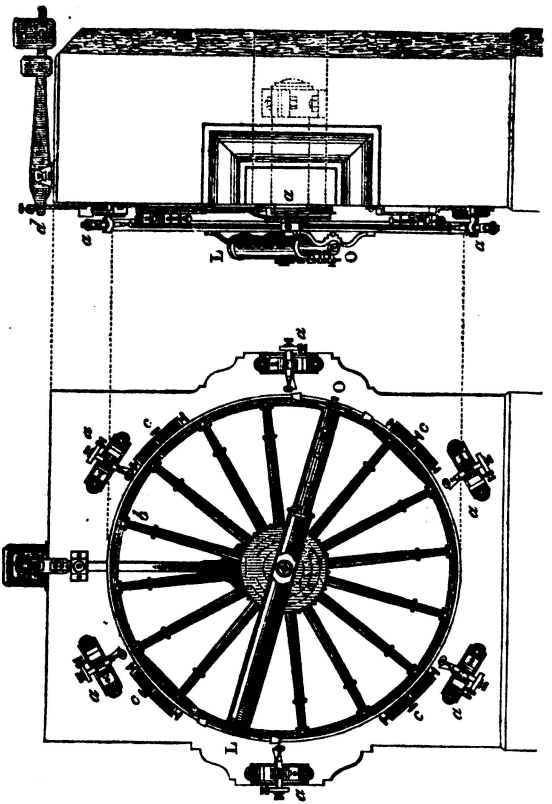
13. **The Mural Circle.**—The mural circle consists, as shown in the annexed figure, of a graduated circle, provided with a telescope, and firmly attached to a wall in the plane of the meridian. It is intended for the measurement of altitudes or zenith distances, from which the declination can be found, when the latitude of the observatory is ascertained. The figure represents a front and side view of the large mural circle of the Paris Observatory.

In this figure O and L are the eye and object glasses of the telescope OL, which turns with the mural circle by gently pressing the extremities of the arms *b*, which are covered with velvet, the finer movements of the circle being accomplished by means of the tangent screws *c*, *c*, *c*, *c*; and the readings of the circle being made by means of the six microscopes *a*, *a*, &c.

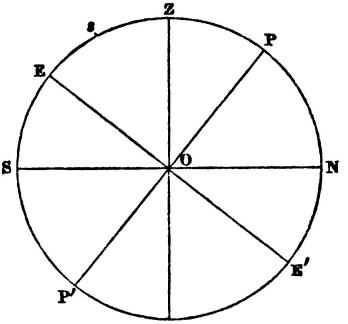
For the purpose of relieving the pressure on the bearings of the axle, a lever *d* with counterpoise weights is made to support part of the weight of the circle.

The zenith distance of an object being found by means of the mural circle, its declination is deduced as follows:—Let the circle SEZPN, page 56, represent the meridian, NOS being the horizon, EE' the equator, Z the zenith, and P, P' the poles.

Let *s* denote a star passing the meridian; at the time of its passage it is observed by means of the mural circle, and the arc Z*s*, or zenith distance measured. It was proved in page 46 of this chapter that PON, or the arc PN, the altitude of the pole, is equal to the latitude of the observatory; but PZ is the complement of PN, and



therefore equal to the colatitude, adding, therefore, to the zenith distance Zs observed, the co-latitude PZ, we



shall have Ps, the polar distance of the star, the difference between which and 90° is its declination.

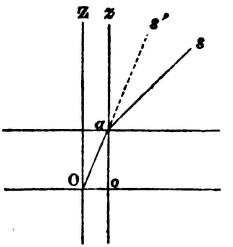
The latitude of the observatory, which is required in order to find the declinations of celestial objects, is found with the aid of the mural circle by measuring the greatest and least altitude of any circumpolar star, and taking the arithmetic mean between these altitudes,—this arithmetic mean is the latitude of the observatory.

14. **Refraction of the Atmosphere.**—The rays of light which come from the stars and other celestial objects to the Earth traverse the atmosphere before they reach the observer at the Earth's surface; and since in traversing the atmosphere they pass continually from a rarer to a denser medium, they will be refracted towards the perpendicular (Manual of Optics). It follows from this consideration, that all the celestial objects appear nearer to the zenith than they actually are, and therefore astronomical observations of altitude or zenith distance require to be corrected for the refraction of the atmosphere.

It is not difficult to prove that for objects within 80°

of the zenith, the correction for refraction is proportional to the tangent of the zenith distance.

In the accompanying figure let O be the position of a spectator, and let *sa*O be the path of a ray of light coming from a star *s*, and refracted at the upper portion of the Earth's atmosphere at *a*. Let ZO and *zo* be vertical lines, then ZO*s'* is the apparent zenith distance of *s*, and *zas* its true zenith distance. Let *z* denote the apparent zenith distance, ZO*s'* = *zas'*, and let the correction for refraction *sas'*, or difference between the true and apparent zenith distances, be denoted by *r*. Then, since *zas* and O*ao* are the angles of incidence and refraction, we have (Manual of Optics),—



$$\sin zas = \mu \times \sin Oao,$$

μ denoting the refractive index of air; hence substituting for the angles their values just given, we obtain

$$\sin (z + r) = \mu \sin z.$$

It is found that *r* is less than 6′, for all zenith distances less than 80°, and therefore its sine may be considered equal to the arc, and its cosine equal unity. Hence, expanding the sin (*z* + *r*), we find,

$$\sin z + r \cos z = \mu \sin z.$$

Dividing both sides by cos *z*, and solving for *r*, we obtain, finally,

$$r = (\mu - 1) \tan z. \qquad (9)$$

The refractive index of air at the temperature 50°, and pressure 29.96, is 1.0002836; and since $r = \frac{r''}{206265}$, r'' being the refraction in seconds, we find,

$$r'' = 0.0002836 \times 206265 \times \tan z = 58''.49 \tan z. \qquad (10)$$

The following Table gives the value of the refraction correction for each degree of altitude, at 50° F., and pressure 29.96 inches:—

TABLE XIV.—*Correction of Altitude for Refraction.*

Altitude.	Refraction.	Altitude.	Refraction.	Altitude.	Refraction.	Altitude.	Refraction.	Altitude.	Refraction.	Altitude.	Refraction.
	33′ 0″	10°	5′ 15″	20°	2′ 35′	30°	1′ 38″	40°	1′ 08″	50°	0′ 48″
1	24 29	11	4 47	21	2 27	31	1 35	41	1 05	55	0 40
2	18 35	12	4 23	22	2 20	32	1 31	42	1 03	60	0 33
3	14 36	13	4 03	23	2 14	33	1 28	43	1 01	65	0 26
4	11 51	14	3 45	24	2 08	34	1 24	44	0 59	70	0 21
5	9 54	15	3 30	25	2 02	35	1 21	45	0 57	75	0 15
6	8 28	16	3 17	26	1 56	36	1 18	46	0 55	80	0 10
7	7 20	17	3 04	27	1 51	37	1 16	47	0 53	85	0 05
8	6 29	18	2 54	28	1 47	38	1 13	48	0 51	90	0 00
9	5 48	19	2 44	29	1 42	39	1 10	49	0 49		

The foregoing Table is the result of observation, and the values of refraction given in it agree very well with the values calculated from equation (10), for altitudes exceeding 10°, or zenith distances less than 80°.

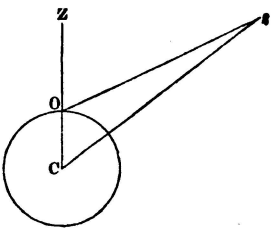
1. Find from equation (10) the correction for refraction of a star at 45° apparent altitude. Here we have $\tan z = 1$, and therefore the refraction $r'' = 58''.49$ differing only $1''.49$ from the observed value given in the Table.

2. Find the refraction of a star at 72° altitude. *Ans.* $r'' = 19''.003$.

15. **Parallax.**—It has been shown in the preceding section that the observed zenith distance of a star differs from its true zenith distance, in consequence of the refraction of light produced by the Earth's atmosphere, the amount of which is given in Table XIV.

Observations of stars made in different places differ from each other also in consequence of the different posi-

tions of the observers; and it is customary to reduce the observations made in any observatory to the observations which would be made by an observer at the centre of the Earth, if such were possible.



This is called correcting for parallax, and is effected as follows:—

Let O be the position of an observer, s a star, C the centre of the Earth, and COZ the line drawn from the centre of the Earth through the place of the observer to the zenith Z; ZOs is the observed zenith distance of the star s; ZCs is the zenith distance of the star s as seen from the centre of the Earth, and the difference of these zenith distances, which is equal to the angle CsO, is called the parallax of the star s.

Parallax may therefore be defined as *the angle subtended at a celestial object by the radius joining the place of an observer with the centre of the Earth.* Its magnitude is found as follows:—Let the zenith distance ZOs be called z, the parallax OsC be called p, and the radius of the Earth OC and distance Cs be denoted by r and R respectively. Then in the triangle OCs (Manual of Trigonometry),

$$\sin \mathrm{O}s\mathrm{C} : \sin \mathrm{CO}s :: \mathrm{OC} : \mathrm{C}s,$$

or,

$$\sin p : \sin (180^\circ - z) :: r : \mathrm{R}.$$

Hence,

$$\sin p = \frac{r}{R} \sin z. \qquad (11)$$

When the celestial object is in the horizon, the triangle COs becomes right-angled at O, and the parallax is then a maximum, and equal to half the apparent diameter of the Earth seen from the point s; in this case it is called

the horizontal parallax, and denoted by P, which is found from the equation,

$$\sin P = \frac{r}{R}. \qquad (12)$$

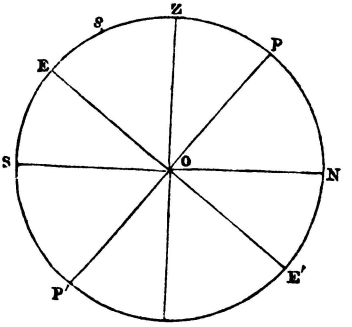
Equations (11) and (12) are used to find the distances R of celestial objects, when their parallax is known, and also the radius of the Earth r, the magnitude of which is given in section 10. This method of finding the distances of celestial objects will be explained in the next chapter, and used to find the distance of the Moon from the Earth.

16. **To find the Latitude of a Place on the Earth's Surface.**—It has been proved in section 9, that the latitude of a place on the Earth's surface is equal to the altitude of the pole. If, therefore, the altitude of the pole could be ascertained, the latitude of the place would be known. A method of finding the altitude of the pole, and, consequently, the latitude, by means of the mural circle was mentioned in page 54, but as this method is only suited to astronomical observatories, it is necessary to describe here the method of finding the latitude used by sailors and travellers.

It consists in observing, by means of Hadley's sextant (Manual of Optics, Chap. IV.), the altitude of the Sun on passing the meridian; which is done at sea by measuring the maximum altitude of the Sun above the sea horizon at noon; and where there is no sea horizon, by measuring the angle between the Sun at noon and his image reflected from a basin of mercury.

The altitude of the Sun at noon being determined, the latitude is thus found. Let s denote the Sun passing the meridian PZES at noon, and let us suppose that he is north of the equator EE'. The Sun's altitude sOS at noon is given by observation, and the Sun's declination sOE is given for each day of the month on pages i. ii. of the Nautical Almanack. The difference between these angles, or EOS, is equal to the zenith distance of the

pole ZOP (because EOP and ZOS are each 90°); and is, therefore, equal to the colatitude.



If the Sun be south of the equator, the sum of his altitude and declination will be equal to the colatitude. Hence we have in general,

$$\text{Altitude} \pm \text{Declination} = \text{Colatitude.} \qquad (13)$$

The sign + is to be used from 22nd September to 20th March, and the sign − from 20th March to 22nd September.

As an example of the foregoing method, the following case may be taken. In the first column is given the times of observation, according to the observer's watch; and in the second column the angle between the Sun and its image reflected in mercury.

Latitude of Trinity College Park, 17th April, 1852.

Time.	Angle between the Sun and Image.
$11^h\ 55^m\ 0^s$	$94°\ 34'\ 40''$
12 0 30	94 37 40
12 4 15	94 39 40
12 5 6	94 40 20
12 7 20	94 39 0
12 10 6	94 37 0
12 13 0	94 33 20

From these observations it appears that the greatest angle between the Sun and his image on that day was 94° 40′ 20″, and half this angle, or 47° 20′ 10″, will be the apparent meridian altitude of the Sun; but from the Table of Refraction, p. 58, this altitude is too large by 52″, and therefore the true altitude is 47° 19′ 18″; but the Sun's declination at noon of that day was 10° 38′ 11″ N., which, subtracted from the altitude, gives 36° 41′ 7″ for the colatitude, and therefore,—

Latitude of Trinity College Park = 53° 18′ 53″.

17. **To find the Time at any Place on the Earth's Surface.**—From the moment that a fixed star crosses the meridian of any place until it returns to the meridian again is $23^h\ 56^m\ 4^s$; but from the apparent motion of the Sun from day to day in the ecliptic, the interval between two meridian passages of the Sun is always greater than that of a fixed star; this interval from noon to noon is variable, partly on account of the obliquity of the ecliptic and partly on account of the unequal motion of the Earth in her orbit, and consequently of the Sun in the ecliptic; the average length of the interval from noon to noon is 24^h of ordinary time, called mean solar time to distinguish it from sidereal time, explained in p. 53. It is evident that the Sun in his apparent diurnal movement from east to west will pass successively over the meridians of different places, and that noon will occur later at places on the Earth's surface according to their longitude west of any given meridian.

Let us suppose the meridian of Greenwich chosen as our standard; then the Sun will cross the meridian of a place 15° west of Greenwich one hour later than it crossed the Greenwich meridian, i. e. at one o'clock, Greenwich time, and similarly for other places; time being converted into longitude at the rate of 15° per hour, which is 360° per 24 hours.

The time of apparent noon is the time of the Sun's meridian passage, and may be found from the observations which give the latitude, because the time of meridian passage is also the time of greatest altitude. The time of apparent noon is best found by noting the times

of equal altitudes before and after noon, and bisecting the interval between them.

As the average interval from noon to noon is 24 hours, and as the Sun is sometimes on the meridian before, and sometimes after, the mean time of passage, tables have been constructed for each day of the year, showing the difference between apparent and mean noon. All common clocks and watches are constructed for mean solar time, or 24 hours per day, and at 12 o'clock each day mean noon occurs; the difference between mean and apparent noon is called the *equation of time*, and is given in pp. i. ii. of the Nautical Almanack for each month. When the time of apparent noon has been found as just described, it is to be corrected for the equation of time, and the result will be the mean local time of the place.

18. **To find the Longitude of a Place on the Earth's Surface.**—In the preceding section we have explained the manner of finding the mean solar time at any place on the Earth's surface, and shown that it is later or earlier than the time of a fixed meridian, such as Greenwich, according as the place considered is situated to the west or to the east of the fixed meridian. It has also been explained that 15° difference in longitude corresponds to one hour of difference in time.

All the methods of finding the longitude of a place on the Earth's surface depend on the fact just stated. We must determine the local time accurately, and compare it by some method with Greenwich time, and then convert the difference of time into degrees at the rate of 15° per hour of time. The whole difficulty of the problem, it will be seen, depends upon the finding of the Greenwich time, as the local time may be readily found by the methods described in the preceding section. The following methods have been used to find the Greenwich time at any moment:—

1st. *To find the Greenwich time by Chronometers.*—The most obvious of all methods of finding the time at Greenwich is to set a good watch or chronometer by the

Observatory clock at Greenwich, and refer to it during the voyage for the true time at Greenwich. This simple method is in common use among sailors, but is liable to the errors unavoidable in the construction of chronometers, which can only be depended upon within certain limits.

2nd. *Lunar Method of finding Greenwich Time.*—This method consists in using the Moon as the index hand of a chronometer, and the stars as the figures of the dial plate. The details of the method are unsuitable to an elementary manual, but the principle of it is easily understood. The distances of the Moon from various stars, near to which she passes in her course through the sky, are given in Greenwich time in a table in the Nautical Almanack, for every third hour of the day; these distances are corrected for parallax and refraction, i. e. they are such as would be seen, undisturbed by Refraction, by a spectator placed at the centre of the Earth. This table of lunar distances may be used as a clock to find the Greenwich time as follows:—The spectator at any place should observe, with Hadley's sextant, the distance from the Moon to one of the stars mentioned in the Almanack; having done so, he has only to correct this distance for refraction and parallax, as explained in sections 14, 15, by which means he finds the true lunar distance as it would be seen from the centre of the Earth; referring then to the Nautical Almanack, he will find written opposite to the lunar distance the corresponding time at Greenwich; and thus finds the Greenwich time. He already knows, by his day observations, the local time, and by converting the difference of local and Greenwich times into degrees at the rate of 15° per hour, he knows his longitude east or west of Greenwich, according as the local time is earlier or later than the Greenwich time shown by the Moon. This method of finding the longitude is more certain than the method by chronometers, on account of the infinite skill and wisdom with which the celestial clock has been constructed; but as it requires more practice and know-

ledge in the observer, it is consequently not in much repute among our mercantile mariners, who, as a class, "despise lunars."

3rd. *To find Greenwich time by Jupiter's Satellites.*—An excellent method of finding Greenwich time on land, and with good telescopes, is the method known as the finding of the longitude by means of the eclipses of Jupiter's satellites. The times of these eclipses are given in the Nautical Almanack in Greenwich time, and may be observed with a telescope for local time; and the difference of local and Greenwich times can be converted into degrees of longitude in the usual way.

There are other methods of finding the longitude; such as observations of rockets from a series of stations; the electric telegraph connecting astronomical observatories, and so giving to each observatory the local time of the others; and the method practised in fixed observatories, and known as the method of finding the longitude by Moon culminating stars, which consists in determining the Moon's motion in right ascension by a comparison of her transit with that of stars which pass the meridian at nearly the same time with her.

In recent times, the most interesting measurements of longitude were those made in the United States to connect the Cambridge observatory with Greenwich, and so with the rest of Europe. Since the establishment of the Collins line of steamers from New York to Liverpool, each vessel carries out and home a number of chronometers which are carefully compared with the English and American local times, and the results registered in the United States. Simultaneously with these observations, the longitude of the United States observatory has been determined by the method of lunar distances, Moon culminating stars, and eclipses of Jupiter's satellites. The result of this interesting series of observations has not yet been published in full, but it is understood that there are slight differences in the results obtained by the various methods, which require explanation.

The American astronomers are disposed to place most reliance on the difference of longitude found from the chronometers.

19. **The Tides.**—The facts connected with the daily rise and fall of the waters of the sea are not only interesting from their connexion with astronomical science, but are also of the highest practical importance to those engaged in navigation. These facts may be briefly summed up as follows:—On the day of new Moon, the Moon and Sun cross the meridian at the same time, i. e. at noon, and at an interval after their passage, varying according to the place of observation, but fixed and definitè for each place, high water occurs; the water, having reached its maximum height, begins to fall, and after a period of $6^h\ 12^m$ reaches a minimum height; it then rises for $6^h\ 12^m$, and reaches a second maximum, falls for another interval of $6^h\ 12^m$, and rises again during a fourth interval of $6^h\ 12^m$. It has, therefore, two maxima and two minima in a period of $24^h\ 48^m$, which is called a tidal day.

When the Moon is full, she crosses the meridian 12 hours after the Sun, i. e. at midnight, and the phenomena of the tide are the same as at new Moon.

The interval of time from noon to the time of high water is the same on the days of new and full Moon, and is called the *Establishment of the Port.*

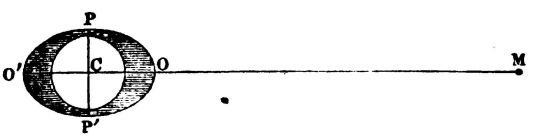
The difference in height between the high and low water is called the range of the tide, and is greatest on the second or third day after new and full Moon, and the tides at these periods of the month are called Spring Tides. When the Moon crosses the meridian at 6 o'clock A. M. or P. M., she is said to be in *quadrature*, and is 90° distant from the Sun.

The tides which occur on the second or third day after the Moon is in quadrature have the smallest range of any tides in the month, and are called *Neap Tides.*

The appearances of the tides which have been just described are explained by the attraction of the Sun and

Moon, the latter of which is most powerful in producing the tide. There are two circumstances connected with this subject which require explanation, viz., why there are two tides each day, and why there are spring and neap tides.

1st. *Cause of the Semidiurnal Tide.*—The tides are caused by the attraction of the Moon or Sun raising the water under the luminary, and there is no difficulty in understanding why the high water should occur at the time, or a short time after, the Moon crosses the meridian. The reason of the second tide which occurs at



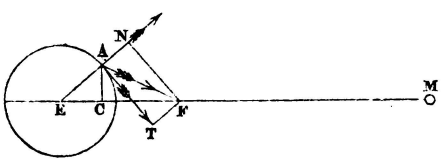
the side of the Earth opposite to the Moon may be understood from the following explanation:—

The attraction of the Moon M upon the centre of the Earth C, and upon the water at O and O', is proportional inversely to the squares of the distances MC, MO, and MO', and is greatest at O, and least at O'; but since the waters of the sea are free to move, they will rise towards the Moon, and move faster towards it than the solid land, which moves at the same rate as the centre of the Earth C; and in like manner the Earth C moves towards the Moon faster than the waters of the sea at O' can follow, and, therefore, the water at O' is left behind, and produces the effect of a rise of tide at O'. The sea is, therefore, drawn out in an elliptical form, having its major axis OO' always directed towards the Moon, and its minor axis PP' at right angles to this line. It is, therefore, evident that there will be high water at O and O', under and opposite to the Moon, and low water at P and P', 90° distant from the Moon. And as the Moon appears to move round the Earth in $24^h\ 48^m$, the ellipsoid of water will follow its motion and also move round the

Earth in $24^h\ 48^m$. There will thus be two high and two low waters each day of $24^h\ 48^m$ duration, corresponding with the moments at which the points O, O′ and P, P′ pass over the place considered.

2nd. *Cause of Spring and Neap Tides.*—The Sun causes a tide in the sea similar to that caused by the Moon, but smaller in amount, and is followed in his apparent path round the Earth each day by an ellipsoid of water. At new and full Moon, the ellipsoids of water caused by the Moon and Sun coincide in direction, and, therefore, the tide is caused by the *sum* of the lunar and solar tides. On the contrary, when the Moon is in quadratures, as she is 90° distant from the Sun, the points O, O′ of the lunar tide coincide with the points P, P′ of the solar tide, and, therefore, there is lunar high water and solar low water at the same instant. The observed tide is, therefore, the *difference* of the lunar and solar tides. Spring tides are, therefore, the *sum* of the lunar and solar effects, and neap tides are the *difference* of these effects.

In order to form an accurate idea of the manner in which the Moon acts upon the water of the sea, causing the lunar tide, it is necessary to examine more particularly the disturbing force, which is due to the difference of the Moon's attraction on the water and on the Earth. To exhibit this force, we shall make use of a theorem which follows readily from the constructions given by Newton in his Lunar Theory, the proof of which, however, is too difficult for an elementary work.



Let M and E denote the Moon and Earth, and A any particle of water on the Earth's surface. Newton's construction is as follows:—*Let* AC *be drawn perpendicular*

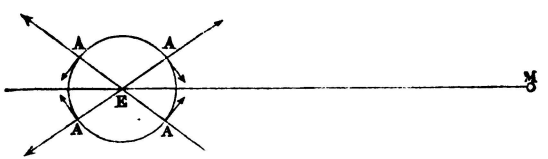
to EM, *and let* EF *be measured off equal to three times* EC, *and let* AF *be drawn. This line* AF *represents, in magnitude and direction, the disturbing force of the Moon* M *upon the water at* A, *the Moon's attraction upon the Earth being represented by the line* ME.

If the disturbing force AF be resolved into AN and AT, in directions coinciding with the radius of the Earth, and tangential to it; AN will represent the part of the Moon's force which lifts the water, and is called the normal component; and AT the part which draws the water towards the line EM, and is called the tangential component. The latter of these components is the most effectual in piling up the water under the Moon, as it acts at right angles to gravity, and is therefore unopposed by it, while the force AN is exerted in direct opposition to the weight of the water, and consequently produces but little effect.

This statement may be better understood by the following figures. The maximum lifting power of the Moon upon the water occurs at the point directly under the Moon, and opposite to it, and is only $\frac{1}{11000000}$th part of the weight of the water; or, more accurately, if g denote the force of gravity,

$$\text{Moon's maximum lifting force} = 0.000000\,1056 \times g.$$

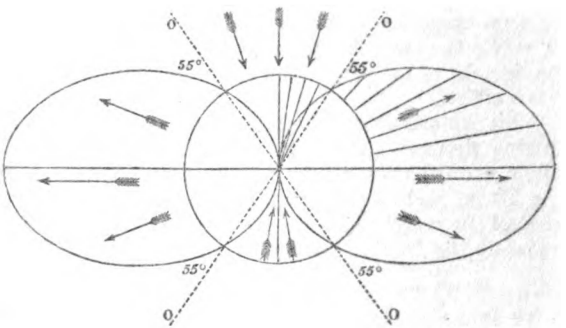
If, therefore, the sea be supposed 10 nautical miles deep, or 10000 fath., this force would only raise the tide $\frac{7}{100}$ths of an inch, which would be absolutely insensible; or, if we suppose it to act upon a cubic fathom of water, weighing 6 tons, its force would be represented by 10 grs.



It is, therefore, to the tangential component AT of the Moon's attraction, that we must look for an explanation

of the tides. This force at its maximum amount is three-fourths of the maximum lifting force, but its whole effect is produced, inasmuch as it acts at right angles to gravity, and is, therefore, unopposed by it. The manner in which it piles up the water at each side of the Earth, under and opposite to the Moon, is shown in the foregoing figure, which represents the normal and tangential force of the Moon upon the sea, at four different points A, A, A, A.

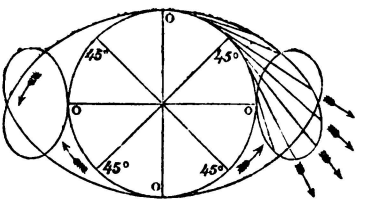
The following figures, which are drawn accurately to scale from the construction given at page 68, show the normal and tangential components respectively. The



first figure is formed by making the construction at page 68, for every point of the circumference of the Earth, and joining the points N; the curve so formed is represented by the double loops shown in the figure, whose branches intersect at the centre of the Earth. This curve intersects the Earth at points shown by the lines OO and OO, 55° distant from the line joining the Earth and Moon; at these points the Moon exerts no lifting force upon the water, and beyond these points the lifting force is converted into a depressing force, which tends to lower the sea, and cause low water at 90° distant from the Moon. This depressing force reaches a maximum at 90° distant from the Moon, and

is then one-half the lifting maximum. The direction of the forces is shown by the arrows in the figure.

The second figure is formed in like manner from the construction in page 68, by joining the points T found



for each part of the Earth. The tangential curve so formed touches the Earth in four points O, O, O, O, at which the tangential component of the Moon's attraction is evanescent; it reaches a maximum at four points, 45° distant from these points, this maximum being three-fourths of the maximum lifting force of the Moon, and always drawing the water *towards* the line joining the Earth and Moon, so as to pile it up under and opposite the Moon, as shown in page 67.

It can be proved, that if the Earth were surrounded with water, as in the figure just referred to, that the lunar tide would rise to a vertical height of 58 in.; and as only $\frac{7}{100}$th of an inch in a sea 10 miles deep is due to the lifting force, it is easy to see what an important part the tangential force plays in the formation of the tide.

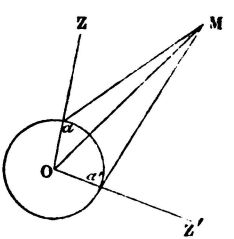
CHAPTER III.

THE MOON.

1. Distance of the Moon from the Earth.—2. Diameter of the Moon. —3. Apparent path of the Moon in the Sky.—4. Elliptical path of the Moon.—5. Phases of the Moon.—6. Lunar Eclipses.

THE Earth is accompanied by her satellite, the Moon, in her annual elliptical path round the Sun, and the Moon itself describes an elliptical orbit round the Earth which occupies one of the foci of this orbit. The excentricity of the Moon's orbit is 5.4844 per cent. of its mean distance, which is 59.96435 times the Earth's equatorial radius, or 237640 English miles. The periodic time of revolution is $27^{d}\ 7^{h}\ 43^{m}\ 11^{s}.5$. The diameter of the Moon is 2153 miles, and its density 0.619, the Earth's density being taken as the unit. The inclination of the Moon's orbit to the ecliptic is $5°\ 8'\ 47''.9$, and she revolves on an axis nearly perpendicular to her orbit in exactly the same time as that in which she completes her revolution round the Earth. The manner in which the facts just mentioned relating to the Moon have been discovered, will be explained in this chapter.

1. **Distance of the Moon from the Earth.**—The mode of finding the distance of the Moon from the Earth is the same as that of finding the distance of the Sun or planets, and the description of it here given is, therefore, applicable to any of the heavenly bodies.



Let M represent the Moon at any time; O the centre of the Earth; a, a' two observatories situated on the same meridian; let Oa and OM, the radius of the Earth and Moon's distance, be denoted by r and R res-

pectively; and let $Za\mathrm{M}$ and $Z'a'\mathrm{M}$, which are the Moon's zenith distances as seen from the two observatories, be denoted by z and z'.

Then by equation (11), the Moon's parallax $\mathrm{OM}a$, at the observatory a, will be

$$p = \frac{r}{R} \sin z,$$

and the parallax $\mathrm{OM}a'$ at the observatory a', will be

$$p' = \frac{r}{R} \sin z'.$$

Adding together, we find

$$p + p' = \frac{r}{R} (\sin z + \sin z').$$

But the angle $p + p'$, or $a\mathrm{M}a'$, is equal to the difference between ZOZ' and the sum of the angles z and z';* and since, if the observatories be situated, one north and the other south of the equator, we have ZOZ' equal to the sum of the latitudes of the observatories, $l + l'$; we obtain

$$p + p' = z + z' - l - l'.$$

Substituting this value in the preceding equation, we find

$$z + z' - l - l' = \frac{r}{R} (\sin z + \sin z'),$$

and finally, solving this equation for R, we obtain

$$R = r \times \frac{\sin z + \sin z'}{z + z' - l - l'}. \qquad (14)$$

It appears from the foregoing investigation, that in order to find the Moon's or any heavenly body's distance from the Earth, it is only necessary to know the Earth's radius, the latitudes of two observatories on the same

* Galbraith and Haughton's Euclid, Book I. Prop. XXXII., Cor. 5.

meridian, and to observe the Moon's zenith distance in these observatories at the same moment of time. These conditions are not easy to fulfil in practice; but it is easy to conceive that corrections may be introduced into the calculation, which will render it the same as if the observatories were rigorously on the same meridian, and the Moon's zenith distance taken at the same moment.

It has been already explained, page 60, that the fraction $\frac{r}{R}$ is the horizontal parallax or maximum value of the parallax: we find, therefore, for the Moon's horizontal parallax,

$$\frac{r}{R} = P = \frac{z + z' - l - l'}{\sin z + \sin z'}. \quad (15)$$

The Moon's horizontal parallax is reciprocally proportional to its distance, and by numerous observations it is known to be nearly $57' 6''$; and since the number of seconds in an arc equal to the radius is 206265, we have

$$\frac{r}{R} = \frac{57' 6''}{206265''};$$

and since r, the radius of the Earth, is 3963 miles, we find, solving for R,

$$R = 238596 \text{ miles.}$$

The distance just found is exact enough to illustrate the method of finding the distance of a heavenly body. The most accurate value of the Moon's mean distance from the Earth is 237640 miles.

2. **Diameter of the Moon.**—The distance of the Moon from the Earth being known, it is easy to find the Moon's diameter. To accomplish this, we have only to observe the Moon's apparent diameter in minutes and seconds, and use the following proportion:—

The Moon's diameter in miles : The Moon's distance in miles : : The Moon's diameter in seconds : The number of seconds in an arc equal to the radius.

The average of many observations on the Moon's diameter gives $31' 8''.8$; and substituting this value in the preceding proportion, we find

$$\text{The diameter of the Moon} : 237640 \text{ miles} \\ : : 1868''.8 : 206265''.$$

Hence,

$$\text{The Moon's diameter} = \frac{237640 \times 1868.8}{206265} = 2153 \text{ miles.}$$

3. **Apparent Path of the Moon in the Sky.**—If the Moon be observed day after day, as she passes the meridian, and her altitude and time of passage be noted by means of the mural circle and transit instrument, the following facts will be ascertained:—

1st. The interval between two successive meridian passages of the Moon is, on the average, $24^h 48^m$.

2nd. The Moon's altitude increases, reaches a maximum, diminishes, reaches a minimum, and again increases to a maximum; the time of passing through these changes being $27^d 7^h 43^m 11^s.5$.

3rd. If we trace out the Moon's path among the fixed stars, we shall find it to lie in a plane which makes an angle of $5° 8' 47''.9$ with the plane of the ecliptic.

In the following Table the time of the Moon's meridian passage at Greenwich, and the corresponding lunar declination, are given approximately for 11 days; and an attentive examination of this Table will illustrate the facts just stated. The Moon's declination is found from the altitude measured by the mural circle in the manner described in page 56:—

TABLE XV.—*Lunar Observations.*

Date.	Meridian Passages.	Declination.	Excess of Lunar Interval above 24h.	Change in Declination.
1853.				
April 12	3h 1m.5 P.M.	20° 26′ 31″ N.		
13	3 49 .0	22 59 8	0h 47m.5	+ 2° 32′ 27″
14	4 38 .5	24 24 42	0 49 .5	+ 1 25 34
15	5 29 .5	24 42 28	0 51 .0	+ 0 17 46
16	6 21 .4	23 42 21	0 51 .9	− 1 0 7
17	7 13 .4	21 23 12	0 52 .0	− 2 19 9
18	8 4 .9	17 48 22	0 51 .5	− 3 34 50
19	8 55 .9	13 5 55	0 51 .0	− 4 42 27
20	9 46 .5	7 28 36	0 50 .6	− 5 37 19
21	10 37 .4	1 29 37	0 50 .9	− 5 58 59
22	11 29 .5	4 58 51 S.	0 52 .1	− 6 28 28

From the preceding Table it appears that the excess of the interval between two meridian passages above 24 hours undergoes alteration from day to day, and that the change in declination is different on different days; being most rapid when the Moon's declination vanishes, i. e. when she crosses the equator.

The Moon's path during the 11 days may be laid down on a celestial globe or chart by the figures of the second and third columns of the Table, and its angle with the ecliptic measured.

If the Moon's path were constructed in the sky in the manner just indicated, it would be found to make an angle of about 5° 8′ 48″ with the ecliptic, and it would intersect the latter circle in two points opposite to each other, which points of intersection are called the Moon's *Nodes*. The *Ascending Node* is that point of intersection in which the Moon passes from the south to the north side of the ecliptic; and the *Descending Node* is that point of intersection in which the Moon passes from the north to the south side of the ecliptic.

By ascertaining the position of the Moon's Nodes from

time to time, it has been established that they do not occupy fixed positions on the ecliptic, but that they slowly shift their place, moving from east to west, or contrary to the direction of the apparent motion of the sun in the ecliptic, which is from west to east. For this reason the motion of the Moon's Nodes is called *Retrograde;* and the period in which the Nodes accomplish an entire revolution on the ecliptic is 18.6 years, or at the rate of 3′ 10″.6 per diem.

This movement of the Moon's Nodes, as we shall presently show, is of importance in connexion with the subject of lunar eclipses.

4. **Elliptical Path of the Moon.**—It can be proved, in a manner similar to that illustrated for the Sun in pages 38–40, that the Moon, in her orbit round the Earth, obeys Kepler's first and second laws of planetary motion, i. e.—

1st. The Moon's path is an ellipse, of which the Earth occupies a focus.

2nd. The radius vector of the Moon describes equal areas in equal times.

It is to be observed, however, that the ellipse described by the Moon changes gradually from month to month. The fact that the plane of the Moon's orbit changes its position has been shown by the fact that the *nodes* of the orbit perform a retrograde revolution on the ecliptic in 18.6 years. In addition to this change of position in the orbit, the axis major of the elliptic orbit of the Moon shifts its position gradually in the plane of the orbit, performing a complete revolution from west to east in 8.85 years, or at the rate of 6′ 41″ per diem. The Moon is said to be in *Perigee* when she is nearest to the Earth, and in *Apogee* when she is farthest from the Earth; and the line joining the apogee and perigee is called the line of the Moon's *Apsides;* and the phenomenon just adverted to is called the *Progression of the Lunar Apsides.* The excentricity of the Moon's orbit

also varies within certain limits, being sometimes greater, sometimes less, than its mean value, which is 5.48 per cent.

The excentricity of the lunar orbit may be found by observing the greatest and least apparent diameters of the Moon; and since these diameters are inversely proportional to the distances at perigee and apogee, we have, by equation (6), page 39,

$$\frac{A}{P} = \frac{1+e}{1-e}. \qquad (6)$$

In the month of April, 1853, the greatest diameter of the Moon was observed to be 16′ 43″.1, and the least diameter 14′ 45″.2; find from these data the excentricity of the lunar orbit during that month. By equation (6), we have

$$\frac{1+e}{1-e} = \frac{16'\,43''.1}{14'\,45''.2} = 1.13319,$$

and solving this equation for e, and multiplying the result by 100, to reduce the excentricity to a per-centage of the semiaxis major, we find

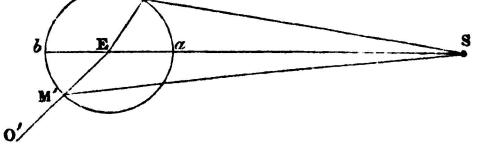
Excentricity of Moon's orbit = 6.24 per cent.

5. **Phases of the Moon.**—Let us suppose the Moon, the Sun, and a known star, to cross the meridian at the same moment on a certain day. At the end of $23^h\ 56^m\ 4^s$ the star, having described its path in the sky from east to west, will be found again on the meridian; the Sun, in the same time, will appear to have gone eastward in the ecliptic about 59′, and will require about 4^m longer than the star to return to the meridian; in the same time the Moon will have travelled eastward in her orbit about 13° 10′ 55″, and will require about 52^m longer than the star, or 48^m longer than the Sun, to return to the same meridian. After a lapse of $27^d\ 7^h\ 43^m\ 11^s.5$, the Moon and star will again have the same right ascension, and be found on the same meridian, the star having passed the meridian of the place of observation once oftener than the Moon. This interval of time is called the Periodic Time of the Moon, or a Lunar Month. The Sun will require $365^d\ 6^h\ 9^m\ 10^s.7$ before he is again on

the same meridian as the star, during which time the star crosses the meridian once oftener than the Sun. If, instead of comparing the Moon and star, or Sun and star, we compare the Moon and Sun, it is plain that when the lunar month has elapsed, and the Moon and star are again on the same meridian, it will require an additional time for the Moon to overtake the Sun, and pass the meridian again in company with him. The interval of time between two successive meridian passages of the Sun and Moon in company is called a *Synodic* Month*, and is found by observation to be equal to $29^d\ 12^h\ 14^m\ 2^s.87$.

The Moon is said to be in *Conjunction* with the Sun when she passes the meridian with the Sun, or has the same right ascension; and since she travels eastward at the rate of 13° 10′ 55″ per day, while the Sun travels eastward at the rate of only 59′ 8″ per day, she will separate from the Sun at the rate of 12° 11′ 47″ per day. In somewhat more than a week, or seven days, she will be 90° east of the Sun, and is then said to be in *Quadrature;* when she is 180° distant from the Sun, she passes the meridian at midnight, and is said to be in *Opposition;* when she has travelled 270° east, or is 90° west of the Sun, she is again in quadrature; and so on.

The hemisphere of the Moon which is turned towards the Sun is enlightened, and the hemisphere turned from



the Sun is dark. In the accompanying figure let E represent the Earth, S the Sun, and *a* the Moon in con-

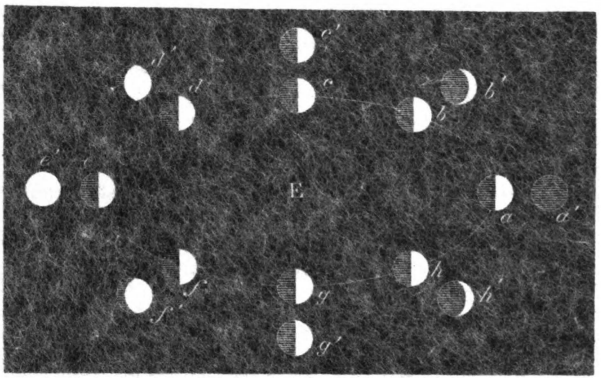
* So called from *συν*, *together*, and *ὁδός*, *a path*.

junction. The dark hemisphere of the Moon is turned towards the Earth in this position, and the Moon is not visible; this condition of the Moon is called New Moon: when the Moon is in opposition at *b*, the hemisphere enlightened by the Sun is turned towards the Earth, and the Moon appears as a full circle of light; this condition of the Moon is called Full Moon: when, however, the Moon is in an intermediate position, M, between conjunction and quadrature, the hemisphere perpendicular to the line MS is enlightened, while the hemisphere perpendicular to the line ME is turned towards the Earth, and it is only the portion common to these two hemispheres that appears bright: this portion is measured by the external angle at the Moon OMS; and as this angle is acute, less than half the Moon will appear bright, and she will therefore be *crescent-shaped*, as shown at *b'* and *h'* in the next figure. When the Moon is at M', between opposition and quadrature, the hemisphere perpendicular to the line M'S is enlightened, and the hemisphere perpendicular to M'E is turned towards the Earth; the enlightened part visible is therefore the part common to these two hemispheres, and is measured by the external angle O'M'S; and since this angle is greater than 90°, more than half of the Moon will be visible, and she will appear of the form shown at *d'* and *f'* in the next figure: the Moon, in this case, is said to be *gibbous*.

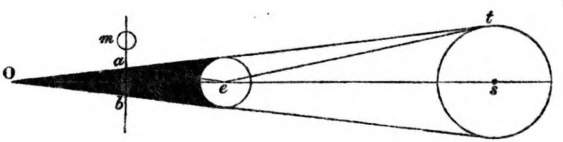
In the annexed figure, E is the Earth, nearly in the centre of the Moon's orbit; and the circles, half light and half dark, represent the Moon enlightened by the Sun, supposed to be placed at a distance on the right-hand of the figure. The figures ranged on the outer curves represent the *Phases* of the Moon as they appear to spectators on the Earth.

The figure at *a'* represents *New Moon* at Conjunction.
" *b'*, *h'* " *Crescent-shaped Moon* between Conjunction and Quadrature.
" *c'*, *g'* " *Half Moon* at Quadrature.

The figure at d', f' represents *Gibbous Moon* between Quadrature and Opposition.
„ e' „ *Full Moon* at Opposition.



6. **Lunar Eclipses.**—The Sun, being a bright object, and larger than the Earth, will form a conical shadow of the Earth in the manner shown in the accompanying figure.



The line *se*O, joining the centres of the Sun, the Earth, and the point of the shadow, will always lie in the plane of the ecliptic; and the axis of the Earth's conical shadow will intersect the ecliptic in a point exactly opposite to the centre of the Sun, or 180° distant in longitude from the Sun. If the Moon performed her revolution round the Earth in the plane of the ecliptic,

she would pass through the Earth's shadow once each month when in opposition, and be eclipsed; but as her true orbit is inclined 5° 8′ 48″ to the ecliptic, it is plain that she may lie above or below the ecliptic from 0° to 5° 8′ 48″ latitude at the time of opposition; and, in fact, she generally passes the shadow of the Earth, when in opposition, in the manner represented in the figure, in which the Moon is shown at *m* passing over the Earth's shadow: in order to undergo an eclipse, she should pass through the disc *ab* drawn through the shadow below her.

In order, therefore, to ascertain the circumstances relating to a lunar eclipse, it is necessary to perform a few simple calculations to determine the length of the Earth's shadow, *e*O, and also the apparent diameter of the disc, *ab*, seen from the Earth.

1st. *To find the length of the Earth's shadow.*—The length of the Earth's shadow could be found if we knew the semi-angle *e*O*t* of the cone. Let *c* denote this angle, and *r* the radius of the Earth; and with O as centre, and O*e* as radius, let a circle be described; we have (Manual of Trigonometry, page 5),

$$Oe : r : : 206265'' : c,$$

or

$$Oe = r \times \frac{206265}{c}. \qquad (a)$$

The angle *c*, or *e*O*t*, is equal to the difference of the angles *set* and *et*O: the first of these is the apparent semidiameter of the Sun; and the second is the semiangle under which the Earth is seen from the Sun, or, as explained in page 59, the Sun's Horizontal Parallax. Let the former of these angles be called *s*, and the latter *p*, and we obtain

$$c = s - p. \qquad (b)$$

Inserting this value of *c* into equation (*a*), we find

$$\text{Length of Earth's shadow} = r \times \frac{206265}{s - p}. \qquad (16)$$

The mean diameter of the Sun, seen from the Earth, is 1923″; and the mean diameter of the Earth, seen from the Sun, is 17″; and therefore $s - p = 953''$; and therefore it follows from equation (16),

that the length of the Earth's shadow is 216.4 times the radius of the Earth: but since the Moon's distance is only 59.9 times the Earth's radius, it follows that the Moon must pass through the shadow when in opposition, provided she do not pass over or under it.

2nd. *To find the angle subtended at the Earth by a section of her shadow made at the distance of the Moon.*—The required angle is double aeO, which is equal to the difference between eat and eOt. Let the required angle aeO be called d; eat will be the Moon's horizontal parallax, which may be called p', or half the angle subtended by the Earth at the Moon; and eOt is half the angle of the shadow already found. Hence we have

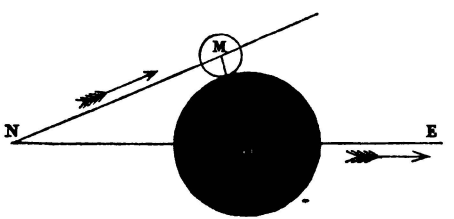
$$d = p' - c.$$

Substituting for c its value in equation (b), we find

$$d = p + p' - s; \qquad (c)$$

or, in other words, half the angle subtended by the section of the shadow is equal to the sum of the parallaxes of the Sun and Moon, minus the Sun's semidiameter.

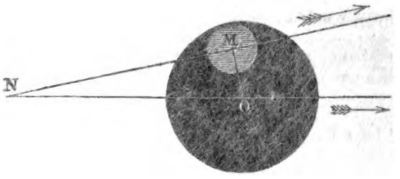
It is plain, from the consideration of the figure, that there will be at least a partial eclipse of the Moon whenever the distance of her centre from the centre of the Earth's shadow is less than the sum of the semiangle of the disc of the shadow, and her own semidiameter. In the annexed figure, let NE represent the ecliptic, NM



the Moon's orbit, and N the ascending node; let the black circle represent the disc of the Earth's shadow, and the light circle the Moon: it is evident that when the line MO, joining the centres of the Moon and shadow, becomes equal to the *sum* of the semidiameters, the Moon will touch the Earth's shadow; and if MO be less

than this limit, the Moon will enter the shadow, and be partially or totally eclipsed, according to the path she describes.

The condition requisite for a total eclipse is shown in the annexed figure, from which it is evident that the arc



MO must become at least as small as the *difference* between the semidiameter of the shadow and the Moon's semidiameter.

The following propositions are easily deduced from the foregoing considerations:—

1st. There can be *no eclipse* of the Moon unless the distance of her centre from the centre of the Earth's shadow become less than the *sum* of the semidiameters of the Moon and shadow.

2nd. There cannot be a *total eclipse* of the Moon unless the distance of her centre from the centre of the Earth's shadow become less than the *difference* of the semidiameters of the Moon and shadow.

3rd. The commencement and end of an eclipse will take place when the distance of the Moon's centre from the centre of the Earth's shadow is equal to the *sum* of the semidiameters of the Moon and shadow.

4th. The commencement and end of the total part of an eclipse will take place when the distance of the Moon's centre from the centre of the Earth's shadow is equal to the *difference* of the semidiameters of the Moon and shadow.

The following problems are given to show the numerous and interesting questions which may be discussed in connexion with lunar eclipses:—

1. *To find the* greatest *apparent magnitude of the disc of the Earth's shadow, formed by a section made at the distance of the Moon.*

The semiangle of the disc, by equation (*c*), is

$$d = p + p' - s. \qquad (c)$$

To make this angle a maximum, we must divide it into two parts p', or Moon's horizontal parallax, which varies inversely at the Moon's distance, and is a *maximum* when the Moon is in perigee; and $p - s$, which is the Sun's horizontal parallax, minus his semidiameter, and is always negative and varies inversely at the Sun's distance; this quantity $p - s$ must therefore be a *minimum*, which it is when the Earth is in aphelion. The maximum and minimum values of these quantities are

Moon in perigee, . . $p' = 60' 13''.7$
Earth in aphelion, . . $p - s = 8''.3 - 15' 45''.$

Adding these values together, we find

Maximum semiangle of disc = $44' 37''$.

2. *To find the* least *apparent magnitude of the disc of the Earth's shadow, formed by a section made at the distance of the Moon.*

To solve this problem, we must take the *minimum* value of p' and the *maximum* value of $p - s$,

Moon in apogee, ... $p' = 53' 58''.3$
Earth in perihelion, . . . $p - s = 8''.7 - 16' 17''.8.$

Adding these quantities, we find

Minimum semiangle of disc = $37' 49''.2$.

The apparent diameter of the Moon in apogee is $29' 24''.7$, and in perigee, $32' 49''.3$; from which it appears that the least diameter of the section of the Earth's shadow is more than double the greatest apparent diameter of the Moon, and consequently that an annular eclipse of the Moon is impossible.

3. *To find when a lunar eclipse* must *take place.*

We have already shown that a lunar eclipse must take place when the distance of the Moon's centre from the centre of the Earth's shadow becomes less than the sum of the Moon's semidiameter and the semidiameter of the shadow; i. e. less than the angle

$$p + p' - s + m,$$

in which m denotes the semidiameter of the Moon. In order, therefore, to have an eclipse inevitable, the distance between the centres of the Moon and shadow must in opposition be less than the minimum value of the foregoing angle; and since the angle of the inclination of the Moon's orbit is small, we may substitute λ, the Moon's latitude, for the distance of the centres, and thus find the condition

λ less than the minimum value of $p + p' - s + m$.

The minimum value of $p + p' - s$ was shown in (2) to be 37′ 49″.2, to which, adding the minimum value of m, in apogee, 14′ 42″.3, we find the Moon's latitude λ less than 52′ 31″.5.

When the Moon's latitude in opposition is less than this quantity, an eclipse *must* happen.

4. *To find when a lunar eclipse* cannot *happen.*

A lunar eclipse cannot happen when the Moon's latitude in opposition is greater than the maximum value of the angle

$$p + p' - s + m.$$

The maximum value of $p + p' - s$, has been shown to be 44′ 37″, and the maximum value of m is 16′ 24″.6; adding these quantities together, we find that a lunar eclipse cannot occur when the Moon's latitude in opposition is greater than 1° 1′ 1″.6.

5. *To find when a total lunar eclipse must happen.*

A total lunar eclipse must happen when the Moon's latitude in opposition is less than the least value of the difference between the angle of the Earth's shadow and the Moon's semidiameter, i. e.

$$\lambda < \text{least value of } p + p' - s - m. \qquad (d)$$

This angle is composed of two parts, $p' - m$ and $p - s$, of which the former is always positive, and varies inversely as the Moon's distance, and the latter is always negative, and varies inversely as the Sun's distance. To make their sum a minimum we must, therefore, have $p' - m$ a minimum, or the Moon in Apogee, and $p - s$ a maximum, or the Earth in Perihelion.

In Apogee, ... $p' - m = 53'\ 58''.3 - 14'\ 42''.3$
In Perihelion, ... $p - s = 8''.7 - 16'\ 17''.8$.

Adding together, we find that a total eclipse must happen when the Moon's latitude in opposition is less than 23′ 6″.9.

6. *To find when a total lunar eclipse is impossible.*

It is easy to see that a total lunar eclipse is impossible when

$$\lambda > \text{maximum value of } p + p' - s - m. \qquad (e)$$

The maximum value of $p + p' - s - m$ will occur when the Moon is in Perigee, and the Earth in Aphelion.

Moon in Perigee, . . . $p' - m = 60'\ 13''.7 - 16'\ 24''.6$.
Earth in Aphelion, . . . $p - s = 8''.3 - 15'\ 45''$.

Hence we find that a total lunar eclipse cannot happen when the Moon's latitude in opposition is greater than 28′ 12″.4.

CHAPTER IV.

THE SUN.

1. Distance of the Sun.—2. Diameter of the Sun.—3. Rotation of the Sun on his Axis.—4. Apparent Annual Motion of the Sun, and its Effects in producing the Seasons.—5. Solar Eclipses.

THE Sun is the most important body in the Solar System, as it is the centre of heat, light, and attraction, to all the planetary bodies which revolve round it. Its mean distance from the Earth is 12032 times greater than the Earth's equatorial diameter; it has a diameter of 888000 miles; and a density one-fourth that of the Earth; and revolves round an axis making an angle of $82° 51'$ with the plane of the ecliptic, in $25^d\ 8^h\ 9^m$.

Many of the effects of the Sun's light and heat upon the Earth, in causing day and night, and the seasons of the year, have been already illustrated in the second chapter on the Earth; and it only remains to develop these facts somewhat in detail, and add a few remarks on solar eclipses, which will be accomplished in the present chapter.

1. **Distance of the Sun.**—The mode of finding the distance of any celestial body was explained in page 73, and consists, as was there shown, in finding the apparent diameter of the Earth as seen from the body whose distance is required. This apparent diameter of the Earth is double the angle called the Horizontal Parallax, which is the maximum parallax of the body, or greatest angle under which the radius of the Earth can be seen.

By accurate observations, made in the manner there described, it is found that the angle under which the Earth would appear to a spectator situated at the Sun is $17''.143$; from which value the Sun's distance may be found as follows:—

The Sun's distance : The Earth's diameter : : The Sun's distance (or radius) in seconds : The earth's disc in seconds;

or

$$\text{The Sun's distance in miles} = \frac{206265''}{17''.143} = 12032.$$

From which we find that the distance of the Sun from the Earth is 12032 times the Earth's diameter.

2. **The Diameter of the Sun.**—The Sun's diameter is found in a manner similar to that of the Moon, by observing his apparent diameter in seconds.

The greatest angle under which the Sun appears is when the Earth is in perihelion, in winter, and is $32' 34''.6$; and the least angle under which the Sun appears is when the Earth is in aphelion, in summer, and is $31' 30''.1$: taking the mean of these two values, we find $32' 2''.35$. From this angle and the angle $17''.143$ subtended by the Earth at the Sun, we can calculate the Sun's diameter as follows:—

The Sun's diameter : The Earth's diameter : : The angle subtended by the Sun at the Earth : The angle subtended by the Earth at the Sun;

or,

$$\text{The Sun's diameter} = 7926^{m} \times \frac{32' 2''.35}{17''.143} = 888791 \text{ miles.}$$

3. **Rotation of the Sun on his Axis.**—Very little, if anything, is known to us respecting the physical constitution of the Sun, and the cause of his continued emission of light and heat. In the absence of positive knowledge on this subject, the conjecture which has obtained most favour among astronomers is the following:—That the Sun is a dark, non-luminous body like the planets, but surrounded with a luminous atmosphere, from which are emitted its light and heat; and that outside this luminous atmosphere there extends another atmosphere not luminous, and resembling, probably, the atmospheres of the Earth and planets. Occa-

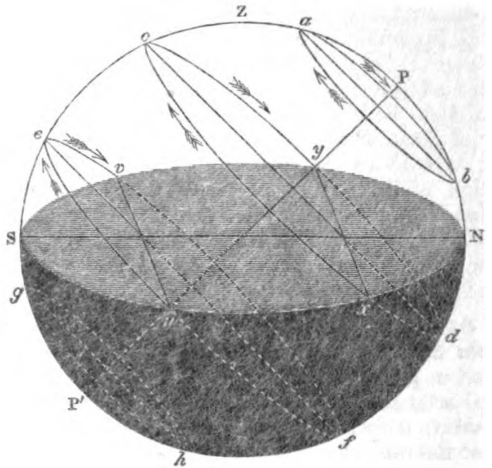
sionally an opening or openings occur in the luminous atmosphere, which are sometimes visible to the naked eye, and present the appearance of a dark centre surrounded by a faint, ash-coloured penumbra: these dark spots are supposed to be the dark body of the Sun, seen through his luminous atmosphere.

Whatever be the cause of these spots, they have enabled astronomers, by watching their apparent motion, to establish the important fact that the Sun has a movement of rotation round an axis, in the same direction (west to east) as the movements of the planets round the Sun and round their axes; which movement of rotation causes the Sun to turn on his axis once in $25^d\ 8^h\ 9^m$.

The equator of the Sun, or plane perpendicular to his axis of rotation, makes an angle of $7^\circ\ 9'$ with the ecliptic.

4. **Apparent Annual Motion of the Sun, and its Effects in producing the Seasons.**—It has been explained in page 37, that the Sun appears to describe an annual orbit in the sky in $365^d\ 6^h\ 9^m\ 10^s.7$; moving in this orbit, which is called the Ecliptic, from west to east at the rate of nearly one degree per diem. It is a consequence of this apparent motion of the Sun, that the solar day is about 4^m longer than the sidereal day. Another and more important consequence of this apparent motion of the Sun is the variation of the seasons, the cause of which has been already explained in page 43. The following consideration of this subject, which is somewhat different, will serve to illustrate it more fully: The difference between summer and winter consists in this,—that during each day in summer the Sun is longer above than below the horizon, while in winter the Sun remains longer below than above the horizon. It follows from this fact, that in summer the ground and atmosphere receive more heat by absorption during the day than they lose by radiation during the night; and, *vice versâ*, more heat is lost at night in winter than is gained during the day.

If the Sun's path, or ecliptic, coincided with the equator, day and night would be of equal duration throughout the year, and there would be no diversity of seasons;



but as the Sun crosses the equator only twice each year, —once in going north, on the 20th March, and once in returning south, on the 23rd September,—it follows that equal day and night occur only twice per annum.

The Sun, having crossed the equator in March, moves northwards until he attains his greatest northern declination on the 21st June. At this period his diurnal path may be represented by the circle *cd*, the greater part of which lies above the horizon. Having recrossed the equator in September, his southern declination increases until the 22nd December, at which period his diurnal path may be represented by the circle *ef*, of which the greatest part lies below the horizon.

The inclination of the ecliptic to the equator, or *obliquity of the ecliptic*, is equal to the maximum declina-

tion of the Sun; it may therefore be ascertained by measuring, by means of the mural circle, the Sun's altitude on midsummer or midwinter day. The greatest declination of the Sun is found to be 23° 28′ both north and south of the equator.

As the altitude of the pole PN changes with the latitude, being always equal to it, the circumstances of the seasons will be different at different parts of the Earth.

1st. *At the equator*, since the latitude is zero, the poles PP′ will lie in the horizon; and all the parallels of declination, *cd*, *ef*, &c., will be equally divided by the horizon; and therefore day and night at the equator will be of equal duration throughout the year.

2nd. *At the pole*, if it were possible for a spectator to be placed in such a position, he would see the pole in his zenith, and the equator would coincide with his horizon. The northern half of the ecliptic would appear always above the horizon, and the southern half of the ecliptic would never rise above it. In such a place there would not be any day or night, properly so called, but the entire year would be divided into two equal portions, during one of which the Sun would continue altogether above the horizon, revolving in circles parallel to it, and during the other portion of the year the Sun would never rise above the horizon.

3rd. *At the latitude* 66° 32′ *north*, the Sun will not set on midsummer day, nor rise on midwinter day. This is easily shown by the following considerations:—

At the latitude 66° 32′ north, the arc PN is equal to 66° 32′, and on midsummer day the Sun's north polar distance is 90° minus 23° 28′, or 66° 32′; therefore on that day the Sun's polar distance will be PN, and he will describe a circle just touching the horizon at the point N. In like manner, on midwinter day, the Sun will describe a circle below the horizon, the upper portion of which will just touch the under side of the horizon at the point S.

4th. *At latitudes less than* 66° 32′ the Sun will always

rise and set, because the altitude of the pole PN, being equal to the latitude, is always less than the least polar distance of the Sun, which is 66° 32′.

5th. *In latitudes greater than* 66° 32′ the year may be divided into three portions: the midsummer portion, during which the Sun never sets; the midwinter portion, during which the Sun never rises; and the intermediate portion, during which he rises and sets as at other parts of the Earth.

The Earth has been divided into five zones, depending on the appearances of the diurnal path of the Sun.

These zones are:—

1st. The Frigid Zones, extending from the North and South Poles to the latitude 66° 32′ north and south. These zones are characterized by the Sun's not rising and setting to their inhabitants during portions of the year.

2nd. The Torrid Zone, extending from the latitude 23° 28′ north, or Tropic of Cancer, to the latitude 23° 28′ south, or Tropic of Capricorn. Throughout this zone the Sun passes through each spectator's zenith twice in the year, when the Sun's declination is equal to the latitude.

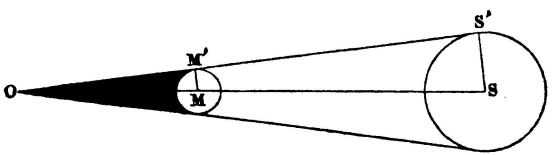
3rd. The North and South Temperate Zones, extending from 23° 28′ to 66° 32′ latitude, and characterized by the following conditions: that in these zones the Sun is never in the zenith, and always rises and sets.

5. **Solar Eclipses.**—In the last chapter we have described the manner in which an eclipse of the Moon occurs, when, in opposition, the dark body of the Earth passes between the Moon and Sun, and throws a shadow on the former; and have explained how a lunar eclipse is possible, because the length of the Earth's shadow is three times the distance of the Moon from the Earth. In like manner, when the Moon is in conjunction with the Sun, her shadow falls upon the Earth; and to a spectator on the Earth the Moon appears to hide the Sun,

either wholly or in part. This phenomenon is called a Solar Eclipse, and would happen once per month if the Moon's orbit coincided with the ecliptic; but as the Moon's orbit is inclined at a small angle to the ecliptic, a solar eclipse will occur only if the Moon, when in conjunction, is also so near her node (or intersection of her orbit with the ecliptic) that the distance between the centres of the Sun and Moon is less than the sum of their apparent semidiameters.

In order to ascertain the peculiarities of solar eclipses it is necessary, in the first place, to ascertain the length of the shadow of the Moon produced by the Sun, which is done as follows:—

Let M and S denote the centres of the Moon and Sun respectively, and O the point of the shadow; it is plain from the figure, that by similar triangles,—



The radius of the Sun, SS' : The radius of the Moon MM' : : OS : OM.

Hence, since the diameters of the Sun and Moon are double their radii, and equal to 888000 miles and 2153 miles respectively, we obtain,

$$OM = OS \times \frac{2153}{888000}.$$

But OS is equal to OM + SM; and SM, the distance of the Sun from the Moon, is the difference of their distances from the Earth, which is at its mean value 95 million miles, or

$$SM = 95{,}000000 \text{ miles.}$$

Hence we find

$$OS = OM + 95{,}000000.$$

Substituting this value in the equation already found, we obtain,

$$OM = (OM + 95{,}000000) \times \frac{2153}{888000}.$$

And solving this equation for OM, we find,

$$OM \times \left(1 - \frac{2153}{888000}\right) = 95{,}000000 \times \frac{2153}{888000};$$

or, multiplying both sides by 888000,

$$OM \times 885847 = 95{,}000000 \times 2153;$$

and finally,

OM = the length of shadow = 230890 miles.

This result gives the average length of the Moon's shadow, which is sometimes greater and sometimes less than 230890 miles. On comparing this result with the distance of the Earth from the Moon, viz., 237640 miles, it appears that they are nearly equal; and, in fact, the length of the Moon's shadow is sometimes greater and sometimes less than her distance from the Earth, according to the varying distances of the Sun and Moon from the Earth.

Eclipses of the Sun are of *three* distinct kinds,—Partial, Annular, and Total.

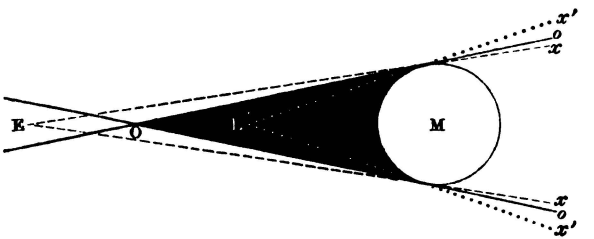
1st. *Partial Eclipse of the Sun.*—An eclipse of the Sun will always occur when the distance between the centres of the Sun and Moon is less than the *sum* of their apparent semidiameters; and this eclipse will be partial unless the distance between the centres be less than the *difference* of the apparent semidiameters.

2nd. *Annular Eclipse of the Sun.*—An eclipse of the Sun will be annular when the distance between the centres of the Sun and Moon is less than the *difference* of their apparent semidiameters, and also the apparent diameter of the Moon less than that of the Sun.

It is easy to show, that in the case of an annular eclipse the point of the shadow does not reach the Earth.

Let M represent the Moon, and O the point of the shadow: the lines O*o*, O*o*, bounding the shadow, are common tangents to the Moon and Sun (as shown in

page 93), and therefore a spectator situated at O would see the Sun and Moon with the same apparent diameters; but a spectator at E would see the Moon under the angle



xEx, less than oOo; and the Moon would therefore appear projected as a black disc on the surface of the Sun, or, in other words, the eclipse would be annular.

3rd. *Total Eclipse of the Sun.*—An eclipse of the Sun will be total when the distance between the centres of the Sun and Moon is less than the *difference* of their apparent semidiameters, and also the apparent diameter of the Moon greater than that of the Sun. In this case, the point of the Moon's shadow reaches and falls inside the Earth, and the spectator is situated at the point E′, where he sees the Moon under the angle x′E′x′, greater than the angle oOo; and the Moon will appear to hide the Sun completely.

CHAPTER V.

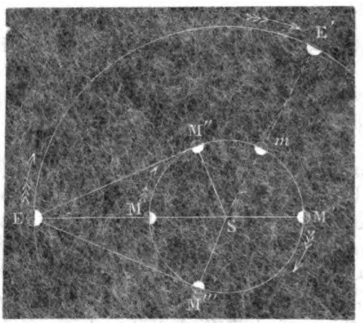
MERCURY.

1. Apparent Path of Mercury in the Heavens.—2. Periodic Time of Mercury.—3. Distance of Mercury from the Sun.—4. Diameter of Mercury.

THE planet Mercury lies nearest to the Sun of all the known planets, being at a mean distance of 0.38710 times the Earth's distance from that luminary. It has a diameter of 3140 miles, and a density 1.12 times that of the Earth. It revolves round the Sun in an orbit, which, compared with those of the other planets, is very elliptical, its excentricity being 20.56 per cent. Its periodic time of revolution is $87^d\ 23^h\ 15^m\ 46^s$, and it has also a rotation on an axis in $24^h\ 5^m$. In consequence of its small size and its apparent nearness to the Sun, it is not easily seen in our latitude; but in more southern latitudes it is often seen, and shines with a brilliancy which procured for it from the observant Greeks the title of the "Sparkling" (ὁ στιλβων). So difficult, indeed, is it to see Mercury in northern latitudes, that it is said that Copernicus, who lived to reach his seventieth year, complained on his death-bed, that, much as he had tried, he had never seen the planet Mercury.

1. **Apparent Path of Mercury in the Heavens.**—The path of Mercury in the sky will be readily understood from the accompanying figure, in which S represents the Sun, MM″M′ the path of Mercury round the Sun, and EE′ part of the path of the Earth round the Sun. If the plane of Mercury's orbit coincided with the plane of the ecliptic, he would move across the Sun's disc as a dark spot at each revolution, when in conjunction with the Sun in the position M; and also would disappear behind the Sun at each revolution, when in

conjunction in the position M′. The positions M and M′ of the planet are called its inferior and superior conjunctions. As the plane, however, of Mercury's orbit makes an angle of 7° with the ecliptic, the planet, when in its



inferior conjunction, will in general pass over or under the Sun, unless it happen to be at one of the nodes of its orbit at the time of inferior conjunction, when the circumstances of the transit of Mercury become similar to those of a solar eclipse, and are computed on the same principles. Let us suppose the planet in superior conjunction at M′; it will appear to a spectator at E to move from M′ to M‴, from west to east; gradually becoming more distant from the Sun, until it arrives at the point M‴, which is called the point of greatest elongation. Having reached this point, it will appear stationary for some time, and at length turn back towards the Sun, moving from M‴ to M, from east to west. Having arrived at inferior conjunction M, it will continue its motion from east to west as far as M″, the other point of greatest elongation. Here it will again be stationary, and then move from M″ to M′, from west to east, as before.

The motion of Mercury is said to be *Direct* from M″ to M‴, i. e. from west to east; it is said to be *Retrograde*

from M‴ to M″, i. e. from east to west; and the planet is said to be *Stationary* at M″ and M‴.

The best idea of the motion of Mercury in the heavens is that of a pendulous motion to and fro at each side of the Sun, the maximum arc of vibration or elongation SEM″ or SEM‴ being 22° 30′; and the times occupied by the Direct and Retrograde swings of the pendulum being unequal, being 72.5 days for the Direct motion M″M′M‴, and 43.5 days for the Retrograde motion M‴MM″.

2. **Periodic Time of Mercury.**—If the Earth were stationary during a revolution of Mercury round the Sun, his periodic time would be easily found by observing the interval from inferior conjunction to the following inferior conjunction; but while Mercury describes his orbit MM″M′M‴M, the Earth has moved from E towards E′, and has reached E′ before Mercury arrives at his next inferior conjunction at *m*. It follows from this consideration, that the interval from conjunction to conjunction is greater than the periodic time by a quantity which has to be determined. This may be accomplished as follows.

Let E and M denote the periodic times of the Earth and Mercury respectively, and let T denote the observed interval from conjunction to conjunction, all these quantities being expressed in days. The number of degrees as seen from the Sun, traversed by the Earth and Mercury per day, are $\frac{360^\circ}{E}$ and $\frac{360^\circ}{M}$ respectively, and the difference of these quantities will be the number of degrees which Mercury gains on the Earth each day; but as in the time T Mercury has gained 360° on the Earth, $\frac{360^\circ}{T}$ will also represent the daily rate of separation, or the gain of Mercury upon the Earth. Hence we find, equating these values,

$$\frac{360^\circ}{T} = \frac{360^\circ}{M} - \frac{360^\circ}{E}. \qquad (17)$$

In this equation E is known, and is $365^d.25637$; T is the interval between two inferior conjunctions of Mercury, and is observed to be $115^d.88$. From these values, M, the periodic time of Mercury, can be calculated as follows.

Dividing both sides of (17) by 360, and solving for M, we find

$$M = \frac{ET}{E+T}; \qquad (18)$$

and substituting for E and T their values, we obtain

Periodic time of Mercury = $87^d.9707$.

The interval $115^d.88$ from one inferior conjunction of Mercury to the next inferior conjunction is called the Synodic Period of Mercury, for the same reason that the interval from conjunction to conjunction, in the case of the Moon, is called the Synodic Month (*vid.* page 79).

3. **Distance of Mercury from the Sun.**—There are three methods of finding the distance of Mercury from the Sun, a brief account of each of which is here given.

1st. *To find the Distance of Mercury by means of his greatest Elongation.*—When Mercury is at his greatest elongation from the Sun, at M″ or M‴, the line EM″ or EM‴, drawn from the Earth to Mercury, is a tangent to his orbit; and the angle SM″E is a right angle. Hence in the triangle SM″E (Manual of Trigonometry),

$$\text{SE} : \text{SM}'' :: 1 : \sin \text{SEM}'';$$

but the angle SEM″, or greatest elongation, is 22° 30′, and SE is $95\frac{1}{4}$ million of miles. Therefore,

Distance of Mercury from the Sun = sin 22° 30′ × $95\frac{1}{4}$ m. miles = 36.451000 miles, or 0.38268 times the Earth's distance.

2nd. *To find the Distance of Mercury by means of his Horizontal Parallax.*—The horizontal parallax of Mercury may be found by the method explained in pp. 73, 74 for the Moon; and since double the horizontal parallax is the angle subtended by the Earth's disc at the planet, the distance of the latter may be found as follows:—

The distance of Mercury from the Earth : the diameter of the Earth :: 206265″ : 2*P*″;

P denoting the horizontal parallax of Mercury, which is found to be 14″.9 when Mercury is in inferior conjunction. Substituting this value for *P*, and 7926 miles for the Earth's diameter, we find

Distance of Mercury from the Earth in inferior conjunction

$$= \frac{206265}{29.8} \times 7926 = 54{,}861000 \text{ miles.}$$

Subtracting this value from 95¼ mil. miles, the Earth's distance from the Sun, we find

Distance of Mercury from the Sun = 40,389000 miles.

3rd. *To find the Distance of Mercury from the Sun from his greatest and least apparent Diameters.*—The diameter of Mercury when in inferior conjunction is 11″.2, and when in superior conjunction 5″.0; which diameters are inversely proportional to his distances at those periods respectively. Hence we find the proportion—

The distance of Mercury from the Earth in superior conjunction : the distance of Mercury from the Earth in inferior conjunction :: 11″.2 : 5″.

If we call x the unknown distance of Mercury from the Sun, expressed in millions of miles, we have—

The distance of Mercury from the Earth in sup. conj. = $95\frac{1}{4} + x$.
The distance of Mercury from the Earth in inf. conj. = $95\frac{1}{4} - x$.

and therefore—

$$95\tfrac{1}{4} + x : 95\tfrac{1}{4} - x :: 11''.2 : 5''.$$

Multiplying the extremes together and the means, and equating the products, we have, solving the resulting equation for x,—

The distance of Mercury from the Sun = 36,454000 miles.

4. **Diameter of Mercury.**—The diameter of the planet Mercury may be found by comparing his apparent diameter in inferior conjunction, when greatest, with his horizontal parallax at inferior conjunction, by means of the proportion :

The diameter of Mercury in miles : the diameter of the Earth in miles : : the diameter of Mercury seen from the Earth in seconds : the diameter of the Earth seen from Mercury in seconds.

The diameter of Mercury in inferior conjunction is $11''.2$, and the horizontal parallax at inferior conjunction is $14''.9$. Hence

$$\text{The diameter of Mercury} = \frac{11''.2}{29''.8} \times 7926 = 2979 \text{ miles.}$$

CHAPTER VI.

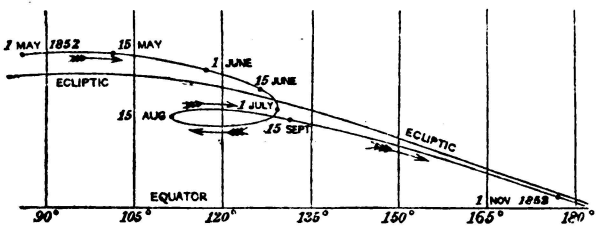
VENUS.

1. Apparent Path of Venus in the Sky.—2. Periodic Time of Venus.—3. Phases of Venus.—4. Distance of Venus from the Sun.—5. Diameter of Venus.

THE planet Venus moves round the Sun in an orbit, which is intermediate between that of Mercury and the Earth. The mean distance of Venus from the Sun is 0.72333 times the Earth's distance, and her periodic time is $224^d\ 16^h\ 49^m\ 7^s$. She approaches nearer to the Earth than any other planet, and resembles the Earth in many respects, having a diameter of 7800 miles, which is about 120 miles less than the diameter of the Earth, and a density of 0.92 times that of the Earth. Venus revolves on her axis in $23^h\ 21^m\ 22^s$, and possesses an atmosphere which appears similar to that of the Earth, and must exert an important influence in modifying the effects of the solar heat and light, which are nearly twice as intense on the surface of this planet as on that of the Earth. Venus is the most beautiful of the planets, and is easily seen in every part of the Earth, as her distance from the Sun is so considerable, that she remains above the horizon, when at her greatest elongation to the east of the Sun, for a considerable period of time; and *vice versâ*, when at her greatest elongation to the west of the Sun, she rises a considerable time before the Sun. In the former case she is called the Evening Star, Hesperus of the ancients; and in the latter, the Morning Star, Lucifer of the ancients. Her light is perfectly white, and differs in this respect from that of the Moon, which is pale yellow, and only appears white in the daytime, in consequence of the yellow light com-

bining with the blue of the Earth's atmosphere. The light reflected from the Sun by Venus when brightest forms a distinct shadow, even in our cloudy latitudes.

1. **Apparent Path of Venus in the Sky.**—A good idea of the apparent path of Venus in the heavens may be formed from an examination of the accompanying



figure, which represents the apparent path of Venus from May 1, 1852, to November 1, 1852.

On the 1st of May Venus was to the north of the ecliptic, and her motion in the sky, as indicated by the arrow, was direct, or from west to east. On the 1st of July she had crossed the ecliptic to the south, and appeared stationary. Her motion then became retrograde until the 15th August, when she was again stationary; after which her motion was again direct, and continued so to the 1st of November, at which time she approached the ecliptic on the south side. This figure, if attentively studied, will convey to the reader a better idea of the actual path of the planet than any lengthened description, as it shows the trace which would actually be left by Venus if we supposed her path to be permanently visible, like that of a burning stick whirled in the air, or a shooting star. This figure also shows the propriety of the term Planets, or Wanderers, invented by the ancients, who were greatly perplexed by the anomalies of such a path as that figured.

2. **Periodic Time of Venus.**—The periodic time of Venus is found in the same manner as that of Mercury, by observing her synodic period, or interval between two inferior conjunctions.

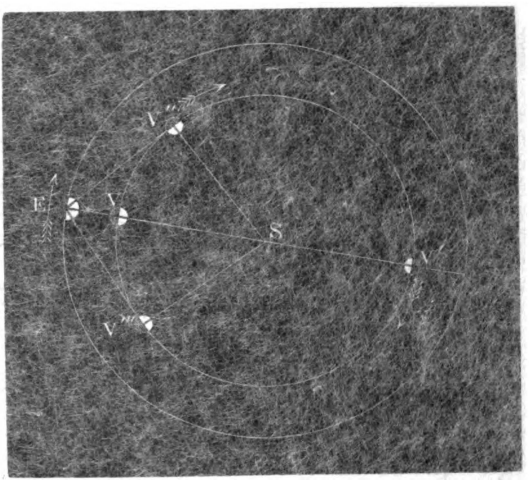
Let T denote this interval, which, in the case of Venus, amounts to 583.5 days. Let E denote the periodic time of the Earth, or 365.25 days; and let V denote the periodic time of Venus. We have by equation (17),—

$$\frac{1}{V} = \frac{1}{E} + \frac{1}{T}. \qquad (19)$$

Or,

$$\frac{1}{V} = \frac{1}{365.25} + \frac{1}{583.5} = \frac{1}{224.7}.$$

Therefore, Periodic time of Venus is 224.7 days.



The accompanying figure, drawn to scale, represents the orbits of Venus and the Earth. V represents Venus in

inferior conjunction, V′ in superior conjunction, and V″, V‴, corresponding to tangents drawn from the Earth E to the orbit of Venus, give the positions of Venus' greatest elongations to the east and west of the Sun respectively. The interval of time from her greatest western elongation, V‴, to her greatest eastern elongation, V″, is shorter in proportion to the whole periodic time than in the case of Mercury, as may be seen from comparing the figure here given with that given at page 97. From the time of inferior conjunction, V, to the next inferior conjunction, is 583.5 days, in which time the Earth has described more than a complete revolution; and consequently Venus, from one inferior conjunction to the next, must traverse 720° + 215°, or more than two circumferences and a half, before she again overtakes the Earth.

3. **Phases of Venus.**—The phases of Venus may be determined by considerations similar to those used in the case of the Moon, page 80. The hemisphere turned towards the Sun is enlightened, and the hemisphere turned towards the Earth would be visible if it were enlightened; from which it follows that the visible portion of Venus is the portion of these two hemispheres which is common.

At inferior conjunction, V, there is no part common to the two hemispheres, and therefore no part visible; the bright hemisphere being turned towards the Sun, and the dark hemisphere towards the Earth.

At superior conjunction, V′, Venus appears like full Moon, because the enlightened hemisphere is turned completely towards the Earth.

At the points of greatest elongation, V″ and V‴, Venus appears like half Moon, because the angles EV″S and EV‴S are right angles. And in like manner it may be shown that Venus is crescent-shaped through the arc V‴VV″, and gibbous through the arc V″V′V‴.

The planet Venus, unlike the Moon, does not shine brightest when full; because at that period she is in

superior conjunction, and therefore most distant from the Earth. As this planet approaches inferior conjunction her diameter appears to increase, but, at the same time, her phase diminishes; and there must therefore be some point at which she will shine most brightly. This point of maximum brightness is found to correspond to an elongation from the Sun of 40° in the superior part of her orbit, the maximum elongation being about 46°.

4. **Distance of Venus from the Sun.**—The distance of Venus from the Sun may be found by three methods, analogous to those employed to find the distance of Mercury from the Sun.

1st. *To find the distance of Venus by means of her greatest elongation.*—When Venus is at her position of greatest elongation from the Sun, the triangle EV″S is right-angled, and therefore $SV'' = ES \times \sin SEV''$ (Manual of Trigonometry). Taking, therefore, the line ES as $95\frac{1}{4}$ million miles, and the greatest elongation at its mean value, 45° 30′, we find

Distance of Venus from the Sun = 67,937000 miles, or 0.71325 times the Earth's distance from the Sun.

2nd. *To find the distance of Venus from the Sun by means of her horizontal parallax.*—The horizontal parallax of Venus, when in inferior conjunction, is half the angle subtended by the Earth's diameter seen from Venus. This horizontal parallax may be found by the method described in pp. 73, 74; and the distance of Venus from the Earth thus found—

The distance of Venus from the Earth : the diameter of the Earth $:: 206265'' : 2P''$;

P denoting the horizontal parallax of Venus, which, at its maximum value, is 29″.6. Substituting this value for P, and 7926 miles for the Earth's diameter, we find

Distance of Venus from the Earth in conjunction = 27,615000 m.
Distance of Venus from the Sun, ... = 67,635000 m.

3rd. *To find the distance of Venus from the Sun by observing her greatest and least apparent diameters.*—The greatest and least apparent semi-diameters of the planet Venus are $28''.5$ and $4''.7$; and if x denote the distance of Venus from the Sun, expressed in millions of miles, we have

$$95\tfrac{1}{2} + x : 95\tfrac{1}{2} - x :: 28''.5 : 4''.7.$$

From which proportion we find—

Distance of Venus from the Sun = 68,281000 miles.

5. **Diameter of Venus.**—The diameter of Venus may be found from that of the Earth by the following proportion:—

The diameter of Venus : the diameter of the Earth : : the greatest apparent size of Venus : twice her greatest horizontal parallax.

$$\text{Diameter of Venus} = 7926 \times \frac{57''.0}{59''.2} = 7631 \text{ miles.}$$

CHAPTER VII.

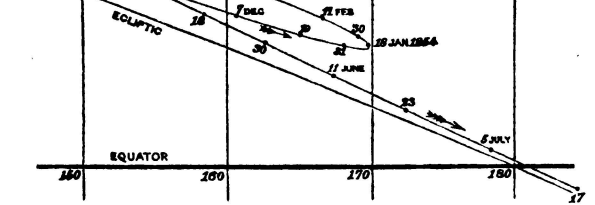
MARS.

1. Apparent Path of Mars among the Stars.—2. Periodic Time of Mars.—3. Phases of Mars.—4. Distance of Mars from the Sun.—5. Diameter of Mars.

THE planet next to Venus, in order of distance from the Sun, is our own residence, the Earth, which has been already fully considered in Chapter II.; we now pass on to the planet Mars, whose orbit lies outside that of the Earth, and which constitutes the outermost member of the remarkable group of interior planets. In addition to the physical division of the planets into Interior and Exterior Planets, they are also divided artificially, with reference to the Earth, into Inferior and Superior Planets; the inferior planets being Mercury and Venus, which lie nearer to the Sun than the Earth; and the superior planets being those which lie outside the Earth's orbit. Mars is the first of the superior planets. He revolves round the sun in an elliptical orbit of 9.33 per cent. excentricity, in $1^y\ 321^d\ 17^h\ 30^m\ 41^s$, at a mean distance from the Sun of 1.52369 times the Earth's distance. The orbit of Mars is inclined to the Earth's orbit or ecliptic at an angle of $1^\circ\ 51'\ 6''$. His equatorial diameter is 4113 miles; the polar diameter being considered by Sir William Herschel as $\frac{1}{16}$, and by Arago as $\frac{1}{30}$, less than the equatorial. He revolves upon his polar axis in $24^h\ 37^m\ 23^s$, and has a mean density 0.958 times that of the Earth. It has been established that there are vast deposits of ice and snow alternately at each pole of this planet, and that these deposits diminish and increase as the pole approaches its summer and winter solstice. This curious fact was observed in 1716 by Maraldi, and first explained in 1784 by Sir W. Herschel; its chief

interest arises from the decisive proof it affords of the presence of considerable quantities of aqueous vapour in the atmosphere of Mars, showing the close analogy between this planet and the Earth, and the probability of the existence of seas and continents on his surface.

1. **Apparent Path of Mars among the Stars.**—The path of Mars, or of any superior planet, differs from the path of an inferior planet in this respect—that, whereas the distance or elongation of the latter from the Sun never exceeds a certain amount, the elongation of Mars or of any superior planet from the Sun may and does become 180°, at which time the planet is said to be in opposition. But, while there is this remarkable difference in the apparent paths of a superior and inferior planet, there is also a striking agreement in the following particular. We have seen, that during a synodic revolution, i. e. from inferior conjunction to inferior conjunction, the apparent path of an inferior planet was,—first, retrograde, then stationary, afterwards direct, and again stationary and retrograde; in like manner the path of Mars, or of any superior planet, in opposition, is retrograde, then stationary, afterwards direct, and again stationary and retrograde, as it a second time approaches opposition.



These facts respecting Mars are well shown by the accompanying diagram, which represents his apparent path

in the sky from 1st of November, 1853, to 17th of July, 1854.

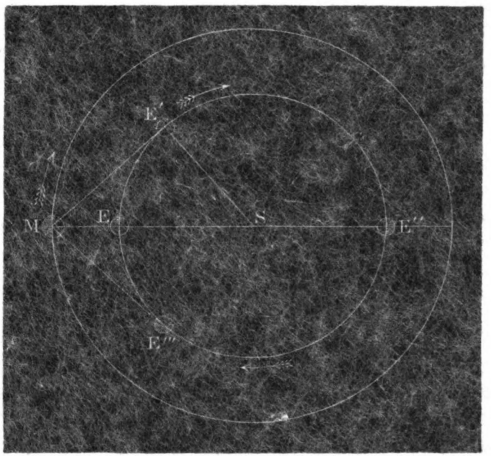
These appearances are explained by the following considerations:—To a spectator on the planet Mars, the Earth would appear as an inferior planet, and would present to him the same phenomena which we witness in Venus or Mercury; viz., retrograde motion in inferior conjunction, direct motion in superior conjunction, with two stationary points intermediate. To a spectator on the Earth viewing Mars, and unconscious of his own motion, these motions are attributed to Mars, on the principle explained, with reference to the Sun, in page 37. Hence Mars will have a retrograde motion in opposition, a direct motion in conjunction, with two stationary points intermediate.

There is this distinction to be observed between the retrograde and stationary appearances of the superior and inferior planets,—that, if the Earth were to stand still there would be no retrograde or stationary appearance in the superior planets; while, on the contrary, the retrograde and stationary phenomena of the inferior planets would still continue; and, although they are modified by, yet they are not the result of, the Earth's motion.

2. **Periodic Time of Mars.**—The periodic time of Mars may be found from observing his synodic time, or interval from opposition to opposition, and knowing the periodic time of the Earth. The calculation is made as follows:—

In the following figure, drawn to scale, the orbits of Mars and of the Earth are represented. When the Earth is at E, Mars is in opposition, and if Mars stood still while the Earth described her orbit EE′E″E‴, Mars would again appear in opposition; but during the Earth's revolution Mars has moved on in his orbit, and the Earth must describe an additional angle in order to overtake him.

Let E denote the periodic time of the Earth, M the unknown periodic time of Mars, and T the observed synodic revolution, or interval from opposition to opposition, all expressed in days; then, $\frac{360^\circ}{E}$ and $\frac{360^\circ}{M}$ will



denote the daily motion of the Earth and Mars, as seen from the Sun, and $\frac{360^\circ}{E} - \frac{360^\circ}{M}$ will denote the rate per diem at which the Earth overtakes Mars; but $\frac{360^\circ}{T}$ also represents the daily rate at which the Earth gains on Mars, and therefore—

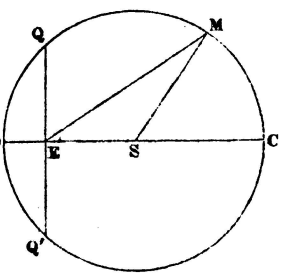
$$\frac{360}{T} = \frac{360}{E} - \frac{360}{M}.$$

Solving this equation for M, we obtain—

$$M = \frac{TE}{T - E}. \qquad (20)$$

The interval T from opposition to opposition is about 780 days, and the Earth's periodic time is $365\frac{1}{4}$ days; substituting these values in equation (20), we find the periodic time of Mars, 687 days.

3. **Phases of Mars.**—In the accompanying figure let OQMC be the orbit of Mars, and E the position of the Earth at any time; when Mars is in opposition at O, his hemisphere perpendicular to SO is enlightened, and is fully turned towards the Earth; therefore, he appears full. Again, when in conjunction at C, he appears full for the same reason. In any intermediate position M, the enlightened hemisphere is that turned towards the Sun, and perpendicular to MS; and the part visible to us, if enlightened, is the hemisphere, perpendicular to ME; but of this hemisphere a portion measured by the angle SME is not illuminated by the Sun, and, therefore, the illuminated portion is measured by the supplement of EMS. It is evident from the figure that EMS is always acute, and, therefore, its supplement always obtuse; and consequently the planet Mars never appears crescent-shaped, but always gibbous or full. It is not difficult to prove that EMS is a maximum at Q or Q′, and, therefore, that Mars looks most gibbous when in quadrature at Q or Q′.



4. **Distance of Mars from the Sun.**—The distance of Mars from the Earth at any period may be found by determining his horizontal parallax, by the method explained in pp. 73, 74. The horizontal parallax of Mars,

which is half the angle subtended by the Earth's disc, to a spectator at Mars, is a maximum when Mars is in opposition, and nearest to the Earth. If the distance of Mars from the Earth in opposition be found, we have only to add to it the Sun's distance from the Earth to find the distance of Mars from the Sun. The horizontal parallax of Mars in opposition is 15″, and therefore :—

The distance of Mars from the Earth : the diameter of the Earth : : 206265″ : 30″.

Consequently the distance of Mars from the Earth in opposition is—

$$\frac{206265}{30} \times 7926 \text{ miles} = 54{,}495000.$$

Adding to this, 95,250000 miles, we find—

Distance of Mars from the Sun = 149,745000 miles.

5. **Diameter of Mars.**—The diameter of Mars is found in a manner similar to that of Mercury and Venus, by the following proportion :—Let P denote the horizontal parallax of Mars in opposition, and D the apparent diameter of Mars at the same period ; then

The diameter of Mars in miles : diameter of the Earth in miles : : D'' : $2P''$;

or,

$$\text{Diameter in miles} = \frac{D}{2P} \times 7926.$$

The maximum apparent diameter of Mars is 15″.5, and the maximum value of P is 15″ ; therefore :—

$$\text{The diameter of Mars} = \frac{15.5}{30} \times 7926 = 4095 \text{ miles.}$$

CHAPTER VIII.

JUPITER.

1. Periodic Time of Jupiter.—2. Distance of Jupiter from the Sun.—3. Diameter of Jupiter.—4. Satellites of Jupiter.—5. Mass of Jupiter compared with that of the Sun.—6. Absolute mass of the Sun.

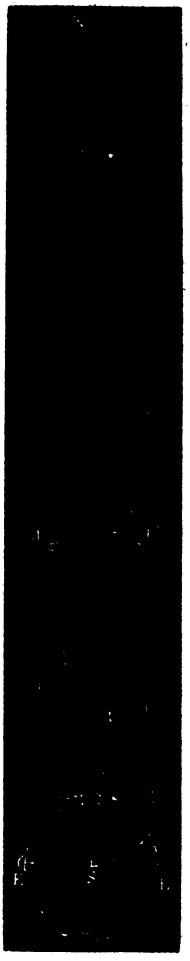
A WIDE interval separates the planet Mars, the outermost of the Interior Planets, from Jupiter, the innermost of the Exterior Planets; this interval being filled partially by the scattered fragments of a solar ring, to which the term Asteroids is applied. We have given in Chap. I. a short account of the Asteroids, and shall now describe briefly the peculiarities of Jupiter and the outer planets. Jupiter is the largest planet in the Solar System, having an equatorial diameter of 89203 miles, and a polar diameter of 82831 miles. He rotates round the latter in $9^h 55^m 21^s.3$, and revolves round the Sun in $11^y\ 314^d\ 20^h\ 2^m\ 7^s$ at a mean distance of 5.20277 times the Earth's distance, in an orbit of 4.81 per cent. excentricity, whose plane makes an angle of $1°\ 18'\ 52''$ with that of the ecliptic. The density of Jupiter is small, as is that of all the Exterior Planets, being only 0.24 times that of the Earth, but his mass is enormous, being to that of the Sun nearly in the proportion of 1 to 1048. The mode of calculating this mass will be explained in the fifth section of this Chapter.

1. **Periodic Time of Jupiter.**—The periodic time of Jupiter is found, like that of Mars, from equation (20). Observation gives the synodic period of Jupiter, from opposition to opposition, as 398.8 days.

Hence the periodic time of Jupiter is

$$J = \frac{TE}{T - E} = \frac{398.88 \times 365.25}{398.88 - 365.25} = 4332 \text{ days.}$$

2. Distance of Jupiter from the Sun.—The method of finding the distance of a planet from the Sun by means of its horizontal parallax, which we have used in the case of Mercury, Venus, and Mars, fails in the case of Jupiter, in consequence of his great distance. The Earth, in fact, subtends an angle of less than 4″ to a spectator on Jupiter, and this is too small an angle to depend on for the accurate determination of distance. In this case, and also for the other Exterior Planets, we adopt a method, depending on what is called the annual parallax of Jupiter, which may be explained as follows: In the annexed figure, which is drawn to scale, let E be the Earth, and J Jupiter, at the time of Jupiter's quadrature, or when the line ES is at right angles with the line EJ; the line JE is then a tangent to the Earth's orbit; let J*e* be drawn also tangential to the Earth's orbit. If Jupiter were not moving, he would appear again in quadrature when the Earth arrived at *e*, and by observing the interval of time from E to *e* we could ascertain the angle ES*e*, and, therefore, EJ*e*, its supplement, which the Earth's orbit subtends at Jupiter. The angle EJS, which is half of EJ*e*, is called the annual parallax of Jupiter, and from its value we could determine the ratio of SE to SJ, as in the case of Venus and Mercury.



But while the Earth moves from E to *e*, Jupiter has also advanced in his

orbit towards J′, and therefore the Earth must pass the point e and go on to E′ before Jupiter again appears in quadrature, or 90° distant from the Sun. When the Earth is at E, Jupiter is said to be in western quadrature, because he is 90° to the west of the Sun; and when the Earth is at E′, Jupiter is said to be in eastern quadrature, because he is then 90° to the east of the Sun. The interval from western to eastern quadrature is found to be $174^d.66$, and as Jupiter advances through $\frac{360°}{J}$ each day; during the interval from western to eastern quadrature he will have advanced

$$\frac{360°}{J} \times 174.66 = 0°.083 \times 174.66 = 14°.30';$$

therefore, the angle JSJ′ is 14° 30′.

Also the angle EXE′ is the supplement of ESE′, which is described by the Earth from quadrature to quadrature; and since the Earth describes $\frac{360°}{E}$ each day, we find—

$$\text{ESE}' = \frac{360°}{365.25} \times 174.66 = 0°.985 \times 174.66 = 172°\ 2'.$$

Therefore EXE′ = 7° 58′.

But in the quadrilateral figure SJXJ′, the sum of the four angles is 360°, and the angles at J and J′ are equal. Subtracting, therefore, from 360° the sum of the angles at S and X, just found, we have, halving the result—

$$\text{SJX} = \frac{360° - 7°\,58' - 14°\,30'}{2} = 168°\ 46',$$

and therefore SJE—

$$\text{Jupiter's annual parallax} = 180° - 168°\ 46' = 11°\ 14'.$$

But, in the triangle SJE (Manual of Trigonometry), the distance of Jupiter from the Sun is

$$SJ = \frac{SE}{\sin 11^\circ 14'}$$

Or,

Distance of Jupiter = 5.·3334 × Distance of Earth.

3. **Diameter of Jupiter.**—The diameter of Jupiter may be found, from knowing his distance from the Earth in opposition, and his apparent diameter in seconds; for, if Δ be the greatest apparent diameter in seconds,

The diameter of Jupiter : The distance of Jupiter from the Earth : : Δ″ : 206265″.

From the preceding section it appears that the distance of Jupiter from the Sun is 5.13 times the distance of the Earth from the Sun, hence, in opposition, the distance of Jupiter from the Earth is 4.13 times the Earth's distance from the Sun. Also the maximum diameter of Jupiter is 45″; therefore,

$$\text{The diameter of Jupiter} = \frac{4.13 \times 95250000 \times 45''}{206265''} = 85823 \text{ miles.}$$

Cassini was the first who observed the difference of length in Jupiter's polar and equatorial diameters; he published his observations in 1691, and gave as the ratio 14 : 15; according to Arago the ratio is 16.7 : 17.7; and according to Hansen and Sir J. Herschel, 13 : 14.

4. **Satellites of Jupiter.**—Jupiter is accompanied in his orbit round the Sun by four satellites or moons, which move nearly in the plane of his equator. Their distances from Jupiter may be found by measuring their greatest elongation, and comparing it with Jupiter's diameter; and their periodic times may also be found by noting the interval between two greatest elongations at the same side of Jupiter, or by observing the interval between two successive eclipses.

It has been found that when Jupiter's diameter is 45″ in opposition, that the greatest elongations of the satellites are—

First Satellite,	136″.10.
Second Satellite,	216.52.
Third Satellite,	345·37.
Fourth Satellite,	607·45.

From which observations are inferred the distances of the satellites, given in the following Table, which contains the most recent determinations of the elements of the Jovian system:

TABLE XVI.—*Satellites of Jupiter.*

No.	Periodic Time.	Distance in Jovian Radii.	Diameter in Miles.	Mass, Jupiter = 1.	Density, Water = 1.
I.	$42^h.47$	6.049	2437	$\frac{1}{57105}$	0.1143
II.	85 .23	9.623	2188	$\frac{1}{23103}$	0.1710
III.	171 .72	15.350	3575	$\frac{1}{11355}$	0.3970
IV.	400 .53	26.998	3059	$\frac{1}{23119}$	0.2275

This Table enables us to make some interesting comparisons between the satellites of Jupiter and the planets. We observe that the masses and volumes of the outer satellites are greater than those of the inner, showing an analogy to the Exterior and Interior Planets; but it is curious to see that the planetary law of densities is reversed, as the outer satellites are the heaviest. It can be also proved, from this Table, that the squares of the periodic times of the satellites are proportional to the cubes of their distances from Jupiter; thus verifying the third law of Kepler for the satellites:—

TABLE XVII.—*Kepler's Third Law.*

No.	T^2	a^3	$\frac{T^2}{a^3}$
I.	1803.7	221.44	8.149
II.	7264.1	891.11	8.152
III.	29488	3616.8	8.153
IV.	160426	19679	8.152

The figures in the last column of this Table are nearly constant, the differences in it being attributable to unavoidable errors of observation. They prove conclusively that the third law of Kepler is true for Jupiter's satellites.

5. **Mass of Jupiter.**—The mass of Jupiter, as compared with that of the Sun, may be found as follows:—

Let R be the distance of Jupiter from the Sun, and T his periodic time, then the centrifugal force of Jupiter in his orbit is

$$4\pi^2 \frac{R}{T^2};$$

but since Jupiter is retained in his path by the attraction of the Sun, which is proportional to the Sun's mass and inversely as the square of the distance (Newton's Law of Gravitation), we therefore have

$$\frac{\text{Mass of the Sun}}{R^2} = 4\pi^2 \frac{R}{T^2}.$$

$$\text{Mass of Sun} = 4\pi^2 \frac{R^3}{T^2}. \qquad (21)$$

For the same reason, if r and t denote the distance and periodic time of one of Jupiter's satellites, its centrifugal force will be

$$4\pi^2 \frac{r}{t^2};$$

and this centrifugal force must be equal to Jupiter's attraction, or

$$\frac{\text{Mass of Jupiter}}{r^2} = 4\pi^2 \frac{r}{t^2},$$

or,

$$\text{Mass of Jupiter} = 4\pi^2 \frac{r^3}{t^2}. \qquad (22)$$

Hence dividing (21) by (22) we obtain—

$$\frac{\text{Mass of Sun}}{\text{Mass of Jupiter}} = \frac{R^3}{T^2} \times \frac{t^2}{r^3}. \qquad (23)$$

The distance of Jupiter from the Sun is 5.20277 times the Earth's distance, which is 2138.07 times the equatorial radius of Jupiter; and, therefore, supposing the radius of Jupiter to be unity;

$$R = 5.20277 \times 2138.07.$$

Also T, the periodic time of Jupiter, is 103982 hours; therefore,

$$\frac{\text{Mass of Sun}}{\text{Mass of Jupiter}} = \frac{(5.20277)^3 \times (2138.07)^3}{(103982)^2} \times \frac{t^2}{r^3};$$

or,

$$\frac{\text{Mass of Sun}}{\text{Mass of Jupiter}} = 127.31 \times \frac{t^2}{r^3}.$$

In this equation, the value of $\frac{t^2}{r^3}$, for each satellite, is the corresponding number in the last column of Table XVII. Substituting these numbers, we find:—

Ratio of masses by 1st Satellite = 1037.5.
" " 2nd Satellite = 1036.4.
" " 3rd Satellite = 1037.9.
" " 4th Satellite = 1037.5.

These values of the ratio of the Sun's mass to Jupiter's mass differ slightly from the more accurate value given in Table I.; this difference arises from neglecting the ellipticity of the orbits, which we have considered, in our calculations, simply as circles, having the Sun and Jupiter placed in their centres.

6. **Mass of the Sun compared with that of the Earth.**—The mass of Jupiter just found is only his relative mass, as compared with that of the Sun; and as we have not yet found the absolute mass of the Sun, it is necessary to do so, in order to complete the determination of the mass of Jupiter. We know, by the methods already explained, the size and density of the Earth, and, there-

fore, its absolute quantity of matter, or mass. If, therefore, we could compare the masses of the Earth and Sun, we should know the absolute mass of the latter, and consequently that of Jupiter, and that of all the planets provided with satellites. Fortunately for the solution of this interesting problem, the Earth is also attended by a satellite, by means of which we can compare the mass of the Sun with the Earth's mass, and so determine the Sun's absolute mass.

Let R and T denote the distance of the Earth from the Sun and her Periodic Time; and let r, t denote the Moon's distance from the Earth and her Periodic Time; then, by the considerations of the preceding section,

$$\frac{\text{The Mass of the Sun}}{\text{The Mass of the Earth}} = \frac{R^3}{T^2} \times \frac{t^2}{r^3}.$$

In this equation,

$R =$ 12032 Equatorial Terrestrial Diameters.
$r =$ 29.982 " " "
$T =$ 365.265 days.
$t =$ 27.321 "

Substituting these values in the preceding equations, we find

$$\frac{\text{Mass of the Sun}}{\text{Mass of the Earth}} = 361584.$$

This number is very close to that given in Table I., viz., 359551; and expresses the number of globes of matter 7926 miles in diameter, and of a specific gravity $5\frac{1}{2}$ times that of water, which are equivalent to the total quantity of matter contained in the Sun. We thus perceive that our own habitation, the Earth, is our standard measure of Weight, as well as of Length and Time; and is the means by which we are enabled to estimate the masses, distances, and dimensions of the heavenly bodies; and also to record the duration of all astronomical phenomena.

CHAPTER IX.

SATURN.

1. Periodic Time of Saturn.—2. Distance of Saturn from the Sun.—3. Diameter of Saturn.—4. Satellites of Saturn.—5. Rings of Saturn.—6. Mass of Saturn.

THE second of the Exterior Planets in order of distance from the Sun is Saturn, which is only second to Jupiter in magnitude, being in its mean diameter 71382 statute miles; its equatorial diameter exceeds its polar in the proportion of 10.2 to 9.2, and it revolves round the latter in $10^h\ 29^m\ 17^s$. Saturn presents a striking analogy to Mars in this respect, that there is a space at each pole, varying in extent and brilliancy according to the seasons of Saturn's year. Whether this increased brightness of the Polar Regions in winter arises from the temporary formation of ice and snow, or be attributed to an unusual accumulation of clouds, it would equally indicate effects due to atmospheric influence and change of season in the planet. Saturn is accompanied in his revolutions round the Sun by eight Satellites and three remarkable Rings, two bright and one darker, which will be described in their proper section. His time of revolution is 29 years, 166 days, 23 hours, 16 minutes, and 32 seconds, and his mean distance from the Sun is 9.53878 times the distance of the Earth from the Sun.

Saturn possesses a less specific gravity than any other known planet, being only 0.14 of the specific gravity of the Earth, or 0.76 of the density of water. The mass of Saturn is $\frac{1}{3501}$ of the mass of the Sun, and is second only to that of Jupiter among the planets.

1. **Periodic Time of Saturn.**—The periodic time of Saturn is found, like that of Mars and Jupiter, by means

of equation (20), from the observation of the interval from opposition to opposition, and the periodic time of the Earth. The synodic period of Saturn is 378.08 days, and, therefore, the periodic time of Saturn,

$$S = \frac{TE}{T-E} = \frac{378.08 \times 365.25}{378.08 - 365.25} = 10763 \text{ days.}$$

2. **Distance of Saturn from the Sun.**—The distance of Saturn from the Sun is found in the same manner as that of Jupiter, page 115, by determining the annual parallax, the sine of which angle gives the ratio of the Earth's distance to that of Saturn. The annual parallax of Saturn is found to be 6° 1′, which gives for Saturn's distance 9.5404 times that of the Earth from the Sun.

3. **Diameter of Saturn.**—The diameter of Saturn may be found by measuring his apparent diameter when in opposition, and knowing his distance from the Earth at that time, by means of the following proportion:—

The diameter of Saturn in miles : the distance of Saturn in miles : : the diameter of Saturn in seconds : 206265″.

Professor Struve found for the greatest diameter of Saturn 17″.991, and the distance of Saturn in opposition is 12032 × 7926 × 8.53878 miles, found by subtracting the Earth's distance from the Sun from Saturn's. Hence

$$\text{Diameter of Saturn} = \frac{17''.991}{206265''} \times 12032 \times 7926 \times 8.53878$$

$$= 70863 \text{ miles.}$$

4. **Satellites of Saturn.**—Saturn is accompanied in his revolution round the Sun by eight satellites, whose periodic times and distances, reckoned in radii of Saturn, are given in the following Table:—

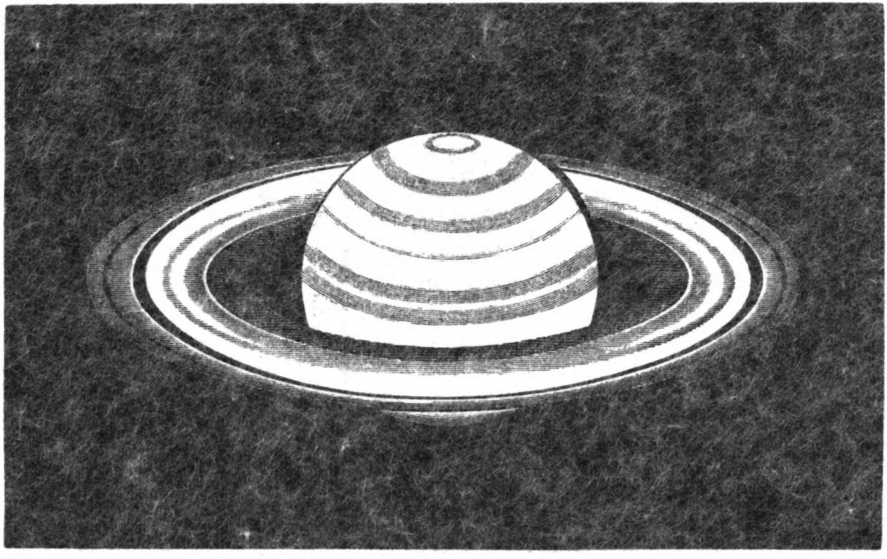
TABLE XVIII.—*Satellites of Saturn.*

Names.	Periodic Time.	Distance in Saturnian Radii.	$\frac{T^2}{a^3}$
Mimas, . .	$0^d\ 22^h\ 37^m\ 22^s.9$	3.3607	13.484
Enceladus, .	1 8 53 6.7	4.3125	13.483
Tethys, . .	1 21 18 25.7	5.3396	13.484
Dione, . .	2 17 41 8.9	6.8398	13.482
Rhea, . . .	4 12 25 10.8	9.5528	13.422
Titan, . .	15 22 41 25.2	22.1450	13.485
Hyperion, .	22 12	28.0000	13.284
Japetus, . .	79 7 53 40.4	64.3590	13.597

In the fourth column of the preceding Table, we have given the ratio of the square of the Periodic Time of each satellite, reckoned in hours, to the cube of its distance from Saturn, reckoned in Saturnian radii. The figures in this column are nearly equal to each other, and this fact proves that Kepler's third law of planetary motion is true in the case of Saturn's satellites.

5. **Rings of Saturn.**—In addition to the eight satellites just described, Saturn is accompanied by a system of thin rings, not more than 140 miles in thickness, and of great breadth, which are more or less luminous, and situated in planes nearly coincident with each other, and with the plane of Saturn's equator. They are represented in the annexed figure, in which are shown two bright rings, the outer apparently divided into two, and an inner ring which is nearly transparent, and reflects light very feebly. This sketch of the rings was made by Mr. Dawes, in November, 1852, at Wateringbury; and another, resembling it in every particular excepting the subdivision of the outer ring, was made by Mr. Lassel at Malta, in December, 1852.

It has been established by the observations of astronomers that these rings are not concentric with Saturn:



Saturn, from a sketch by W. R. Dawes, at Wateringbury, November, 1852.

that they present great irregularities in their surface; and that they revolve with Periodic Times nearly equal to the time of Saturn's rotation on his axis.

The dimensions of the bright rings of Saturn have been determined by Bessel and Struve, as follows :—

External diameter of outer ring	= 40".09 =	176500 miles.
Internal diameter of outer ring	= 35 .29 =	155370 ,,
External diameter of inner ring	= 34 .47 =	151760 ,,
Internal diameter of inner ring	= 26 .67 =	117230 ,,
Distance from inner ring to Saturn	= 4 .34 =	23041 ,,

The mass of the rings of Saturn has been ascertained to be only $\frac{1}{118}$th of the mass of Saturn, from which it has been inferred that the thickness of the rings is only 140 miles, supposing the rings to be composed of materials of the same density as the planet.

6. **Mass of Saturn.**—The ratio of the mass of the Sun to the mass of Saturn may be found by means of his satellites in the manner already described in the case of Jupiter, page 119.

Using the formula (23), we find—

$$\frac{\text{Mass of Sun}}{\text{Mass of Saturn}} = \frac{R^3}{T^2} \times \frac{t^2}{r^3}. \qquad (23)$$

In this equation $\frac{t^2}{r^3}$ is given for each satellite of Saturn in Table XVIII., and $\frac{R^3}{T^2}$ must be expressed for Saturn in Saturnian radii and hours.

Since the Earth's distance from the Sun is 12032 times the Earth's diameter, and Saturn's diameter is 9.022 times the Earth's diameter, we find, referring to Table II.,

$$R = \frac{9.53878 \times 12032 \times 2}{9.022} \text{ Saturnian radii.}$$

$$T = 10759 \times 24 \text{ hours.}$$

Hence we find—

$$\frac{R^3}{T^2} = 247.$$

Using this value of $\frac{R^3}{T^2}$ in equation (23), and introducing successively the values of $\frac{t^2}{r^3}$ from Table XVIII., for each satellite, we find the following values for the ratio of the mass of the Sun to that of Saturn :—

Ratio of Mass of Sun to Mass of Saturn.

From	1st	Satellite,	3330.5
,,	2nd	,,	3330.3
,,	3rd	,,	3330.5
,,	4th	,,	3330.0
,,	5th	,,	3315.2
,,	6th	,,	3330.8
,,	7th	,,	3281.1
,,	8th	,,	3358.4

CHAPTER X.

URANUS.

1. Periodic Time of Uranus.—2. Distance of Uranus from the Sun.—3. Diameter of Uranus.—4. Satellites of Uranus.—5. Mass of Uranus.

THE planet Uranus was discovered accidentally by William Herschel on the 13th March, 1781, during an examination of a small group of stars in Gemini. It is situated at a distance from the Sun of 19.18239 times the Earth's distance, and revolves round the Sun in $84^y\ 5^d\ 19^h\ 41^m\ 36^s$ in an elliptical path of 4.67 per cent. excentricity, and inclined at an angle $0°\ 46'\ 28''$ to the ecliptic. It has a diameter of 34500 miles, and the ratio of its polar to its equatorial diameter is 9 : 10; and, like Saturn, it is accompanied by eight satellites in its path round the Sun.

1. **Periodic Time of Uranus.**—Uranus, being a superior planet, admits of having his periodic time calculated by the formula (20), used for Mars, Jupiter, and Saturn. The synodic period of Uranus is found to be 369.65 days, and since the periodic time of the Earth is 365.25, we find, denoting by U the periodic time of Uranus,

$$U = \frac{TE}{T-E} = \frac{369.65 \times 365.25}{369.65 - 365.25} = 30685 \text{ days},$$

a result which agrees with Table II.

2. **Distance of Uranus from the Sun.**—The distance of Uranus from the Sun might be found by the method used in the case of Jupiter and Saturn; but we prefer to calculate his distance from assuming the truth of Kepler's Third Law, as applied to this planet.

Returning to Table II., which proves the truth of this Law, we find that the ratio of the square of the periodic

time to the cube of the mean distance, expressed respectively in days and distances of the Earth from the Sun, is equal to 133391, taking into the calculation all the principal planets as far as Saturn inclusive. Hence, if U denote the periodic time of Uranus in days, and a his mean distance in terms of the Earth's distance from the Sun, we find, provided Kepler's Third Law be applicable to this planet—

$$\frac{U^2}{a^3} = 133391;$$

and substituting for U its value, 30687 days, and solving for a, we find—

$$a = \sqrt[3]{\frac{(30687)^2}{133391}} = 19.1835;$$

or, the distance of Uranus from the Sun exceeds the distance of the Earth from the Sun in the proportion of 19.1835 : 1. This result agrees so closely with the distance found in the usual way, as to entitle us to consider that Kepler's laws do actually apply to the planet Uranus, a fact which was at one time questioned, from causes which will be explained in the next chapter.

5. **Diameter of Uranus.**—When Uranus is at his mean distance from the Earth, he is found to subtend an angle of 4″, and since in this position (Quadrature) his distance from the Earth is very nearly equal to his distance from the Sun, we have the proportion,—

The diameter of Uranus in miles : distance of Uranus from the Sun in miles : : 4″ . 206265″.

Hence, since the distance of Uranus from the Sun is 19.18239 times the distance of the Earth from the Sun, and this latter is 12032 times the Earth's diameter, we find—

$$\text{Diameter of Uranus} = \frac{19.18239 \times 12032 \times 7926 \times 4''}{206265''} = 35474 \text{ miles.}$$

4. **Satellites of Uranus.**—Uranus is accompanied in his path round the Sun by eight satellites, some of which have only been once observed, in consequence of their requiring the largest telescopes and most favourable weather for observing them. They differ from all other satellites in the following respects,—that they move in orbits inclined to the ecliptic at an angle of 78° 58′, or nearly at right angles; and that their motion is from east to west, or contrary to the motion of all the other bodies of the Solar System, which move from west to east.

Six of the satellites of Uranus were discovered by W. Herschel, who was enabled with his acute power of vision, and the large aperture of his telescope, to observe accurately, with a power of 157, phenomena requiring a power of 300 with ordinary eyes. Of the six satellites discovered by Herschel, five only have been since observed by other astronomers; but there is no sufficient reason to doubt the complete accuracy of W. Herschel's observations.

The following Table contains their distances and periodic times, so far as they have been observed:—

TABLE XIX.—*Satellites of Uranus.*

No.	Periodic Time.	Distance in Uranian Radii.	$\frac{T^2}{a^3}$
1st Satellite, .	$2^d\ 12^h\ 0^m\ 0^s$		
2nd Satellite. .	4		
3rd Satellite, .	5 21		
4th Satellite, .	8 16 56 31.3	17.0	8.882
5th Satellite, .	10 23	19.8	8.911
6th Satellite, .	13 11 7 12.6	22 8	8.808
7th Satellite, .	38 2	45.5	8.868
8th Satellite, .	107 12	91.0	8.833

In the last column of this Table, we have calculated the ratio of the square of the periodic time in hours, to the cube of the distance in Uranian radii, and the constancy of the ratio proves that Kepler's Third Law is applicable to the satellites of Uranus.

5. **Mass of Uranus.**—In order to calculate the mass of Uranus, we use the formula (23),

$$\frac{\text{Mass of the Sun}}{\text{Mass of Uranus}} = \frac{R^3}{T^2} \times \frac{t^2}{r^3}$$

in which equation R denotes the distance of the Sun from Uranus in Uranian radii, T the periodic time of Uranus in hours, and $\frac{t^2}{r^3}$ the ratios calculated for the satellites in Table XIX.

Since the distance of Uranus from the Sun is 19.18239 times the distance of the Earth, which is 12032 times the Earth's diameter; and since the diameter of Uranus is 4.3604 times that of the Earth, we find—

$$R = \frac{19.18239 \times 12032 \times 2}{4.3604} \text{ Uranian radii;}$$

$$T = 30687 \times 24 \text{ hours.}$$

Hence,

$$\frac{R^3}{T^2} = 2187.3.$$

Multiplying this result by the numbers given for $\frac{t^2}{r^3}$ in Table XIX. we find—

Mass of Sun to Mass of Uranus.

From	4th Satellite,	19437
„	5th „	19491
„	6th „	19268
„	7th „	19396
„	8th „	19320

CHAPTER XI.

NEPTUNE.

1. Discovery of Neptune.—2. Periodic Time of Neptune.—3. Distance of Neptune from the Sun.—4. Diameter of Neptune.—5. Satellites of Neptune.—6. Mass of Neptune.

THE planet Neptune is the outermost known planet of the Solar System, and was discovered in consequence of a letter received from Leverrier, of Paris, by Dr. Galle, of Berlin, on the 23rd of September, 1846, stating that he had calculated the position of a planet outside Uranus, the presence of which would account for irregularities in the motion of the latter planet, hitherto unexplained; that the required planet had a longitude of 326°; and asking him to look for it. Dr. Galle, assisted by Encké, did look for it, and found it the same night, with a longitude of 326° 52′. In the first section of this chapter an attempt is made to explain in a general way the manner in which Leverrier was enabled to calculate the position and infer the existence of the planet Neptune.

Neptune revolves round the Sun in a period of $164^{y}\ 226^{d}$, at a distance from the Sun equal to 30.03628 times the Earth's distance, in an orbit whose excentricity is 0.87 per cent. This planet has a diameter of 41500 miles, and a density 0.230, as compared with that of the Earth, and is accompanied by two satellites, the orbit of one of which has been determined.

1. **Discovery of Neptune.**—The planet Uranus was discovered in 1781 by William Herschel, and from the observations made on it, its path was calculated, allowing for the disturbing action of Jupiter and Saturn. By the calculation of its path, it was possible, not only to

predict its place in the heavens at any given future period, but also to assign its place at any past epoch. In this way it was found that the planet Uranus had been observed six times by Flamsteed, as a fixed star, viz.: once in 1690, once in 1712, and four times in 1715; once by Bradley in 1753, once by Mayer in 1756, and twelve times by Lemonnier between 1750 and 1771. Now, although these observations agreed pretty well with the computed place of Uranus, yet there were slight discrepancies, which were attributed to the imperfect observations of the early astronomers. These discrepancies are shown in the following Table:—

TABLE XX.—*Ancient Observations of Uranus.*

Year.	Observed, minus Computed Longitude.
1690	+ 61".2
1712	+ 92 .7
1715	+ 73 .8
1750	− 47 .6
1753	− 39 .5
1756	− 45 .7
1764	− 34 .9
1769	− 19 .3
1771	− 2 .3

It is plain, from an examination of the foregoing Table, that these discrepancies cannot be the result of imperfect observation, as they evidently follow a definite law. This is rendered still more apparent from the following Table, which shows the difference of the observed and computed longitude of Uranus, according to observations made after his discovery:—

TABLE XXI.—*Modern Observations of Uranus.*

Year.	Observed, minus Computed Longitude.	Year.	Observed, minus Computed Longitude.
1780	+ 3".46	1813	+22".00
1783	+ 8 .45	1816	+22 .88
1786	+12 .36	1819	+20 .69
1789	+19 .02	1822	+20 .97
1792	+18 .70	1825	+18 .16
1795	+21 .38	1828	+10 .82
1798	+20 .95	1831	− 3 .98
1801	+22 .21	1834	−20 .80
1804	+24 .16	1837	−42 .66
1807	+22 .07	1840	−66 .64
1810	+23 .16		

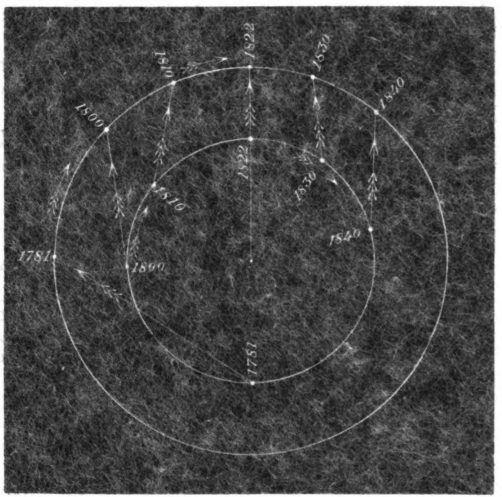
It is evident from the preceding Table that some cause was in operation, not taken into account in the computed place of the planet, tending to increase its longitude from 1781 to 1830, and after that time tending to diminish it.

It was supposed by some that the law of gravitation did not hold rigorously in the case of Uranus; and by others, among whom were Alexis Bouvard and Bessel, that the discrepancies might be accounted for by the action of an unknown planet lying outside the orbit of Uranus.

About the same time two astronomers, M. Leverrier, of Paris, and Mr. John Couch Adams, of Cambridge, attempted to determine the place and magnitude of a planet outside Uranus, which would account for the irregularities of this motion; both succeeded in their attempt, and M. Leverrier first published to the world his results, and led to the actual discovery of Neptune by Dr. Galle, as already described.

The accompanying diagram, which is drawn carefully to scale, shows the paths of Uranus and Neptune from

1781 to 1840, and will illustrate the direction of the perturbing action of Neptune on Uranus.



In 1822 the two planets were in conjunction, at which period the only effect of Neptune's attraction is to draw Uranus farther from the Sun, but not to alter his longitude, because the attracting force, indicated by the arrow, is at right angles to the motion of Uranus.

From 1781 to 1822, Uranus described nearly a semicircle; in the same time Neptune described nearly a quadrant; and throughout the whole of that period, it is evident, from the direction of the arrows, that Neptune tends to draw Uranus in advance of his place, computed by neglecting Neptune's existence.

Again, from 1822 to 1840, the attraction of Neptune is exerted to retard Uranus, and draw him behind his computed place. In 1830, the observed and computed places agree, showing that for eight years from 1822 to

1830, the planet Neptune was employed in destroying the excess of longitude accumulated from 1731; and after 1830 the error in longitude changes sign, and Uranus is kept behind his computed place by the attraction of Neptune.

The problem attacked and solved by Leverrier and Adams is the following, the general possibility of which may be conceived from the figure:—

Given the perturbations produced in Uranus by the action of an unknown planet; required to assign the elements of a planet capable of producing these perturbations.

2. **Periodic Time of Neptune.**—The distance of Neptune from the Sun is so great that the Earth may, without sensible error, be considered as placed in the centre of his orbit, and the motion of Neptune seen from the Earth be considered the same as his heliocentric motion. It is found that Neptune only advances through 2°.187 in a complete revolution of the Earth, and consequently his

$$\text{Periodic Time} = \frac{360^\circ}{2.187} = 164.61 \text{ years.}$$

3. **Distance of Neptune from the Sun.**—The distance of Neptune from the Sun may be found from his Periodic Time, by using Kepler's Third Law. Let T be the Periodic Time, and a the mean distance of Neptune; we find by Table II., from Mercury to Saturn inclusive,

$$\frac{T^2}{a^3} = 133391;$$

Hence,

$$a = \sqrt[3]{\frac{(60126.71)^2}{133391}} = 30.0379;$$

or the distance of Neptune from the Sun is 30.0379 times the Earth's distance.

4. **Diameter of Neptune.**—The apparent diameter of Neptune, when greatest, or in opposition, according to

Encké, is 2".70, and at this period its distance from the Earth is 29.03680 times the Earth's distance from the Sun. But,

The diameter of Neptune : his distance from the Earth :: 2".70 : 206265".

Hence,

$$\text{The diameter of Neptune} = \frac{2''.70}{206265''} \times 29.03680 \times 12032 \times 7926 \text{ miles} = 36247 \text{ miles.}$$

5. **Satellites of Neptune.**—Two satellites of Neptune were observed by Mr. Lassell, one of which only has been seen again by other observers. Its Periodic Time is $5^d\ 21^h\ 7^m$; the inclination of its orbit to the ecliptic $34^\circ\ 7'$; its distance from the centre of Neptune 248850 miles. The existence of this satellite enables us to calculate the mass of Neptune as follows.

6. **Mass of Neptune.**—The ratio of the Sun's mass to the mass of Neptune is found in the same manner as for the planets Jupiter, Saturn, and Uranus—

$$\frac{\text{Mass of Sun}}{\text{Mass of Neptune}} = \frac{R^3}{T^2} \times \frac{t^2}{r^3}.$$

In this equation—

$R = 30.03680 \times 12032 \times 7926$ miles.
$r = 248850$ miles.
$T = 60126.7$ days.
$t = 5.88$ days.

Substituting these values, we find—

$$\frac{\text{Mass of Sun}}{\text{Mass of Neptune}} = 14586.$$

CHAPTER XII.

COMETS.

1. General Phenomena accompanying Comets.—2. True dimensions of Comets.—3. Classification of Comets.—4. Comets of Short Period.—5. Comets of Long Period.—6. Elliptic Comets beyond the limits of the Solar System.—7. Hyperbolic Comets.—8. Parabolic Comets.

In addition to the planetary bodies described in the preceding chapters, our Sun is the centre of attraction to a great multitude of nebulous or gaseous masses, which, under the name of Comets, describe orbits under the influence of his attraction. Of these remarkable bodies, upwards of 600 have been observed from the commencement of the Christian era to the middle of the present century. Of these 600 Comets, the paths of upwards of 200 have been calculated; and of this number, about 40 have been ascertained to move, like the Planets, in elliptic orbits, and, therefore, to revisit the Sun periodically; while more than 160 describe hyperbolic or parabolic paths; and, having once approached the Sun, and receded from his influence, never again revisit the Solar System. In the present chapter we propose to give a brief account of the most remarkable of these bodies.

1. **General Phenomena accompanying Comets.**—When a Comet is visible to the naked eye, it consists generally of a more or less condensed gaseous or nebulous mass, termed the *Head*, containing frequently in its centre a bright point, called the *Nucleus;* and a train of less dense nebulous matter stretching away from the direction of the Sun, and called the Comet's *Tail*. The light emitted by comets proceeds principally from the Head; and it has been shown by delicate optical experiments, that this light is derived by reflexion from the Sun, and

that the Comet is not self-luminous. Some Comets have been so bright as to be visible at noonday, and to cast a sensible shadow at night. The colour of the light is generally white, and has been particularly attended to by the Chinese observers. Of 49 instances recorded by them, the colour was *white* in 23, *bluish* in 20, *red* in 4, and *greenish* in 2 cases. Comets are almost always accompanied by tails, which, as has been already mentioned, are placed in the line joining the Sun and Comet, and on the side opposite to the Sun. Exceptions to this rule, though rare, sometimes occur. For example, the tail of the Comet of 1577 deviated 21° from the line joining the Sun and Comet, and the tail of the Comet of 1680, near 5° from the same line. Comets have been occasionally observed with two tails, one in the usual position, and the other in nearly an opposite direction, or towards the Sun. The angle between the two tails, when such a phenomenon has been observed, has always been very considerable, varying from 140° to 170°. This rare phenomenon of two tails is supposed to be connected with certain rapid changes which the gaseous substance of the Comet is observed to undergo on approaching the Sun. There are many instances on record, in which the tails of comets were observed to stretch through 100° of the celestial sphere, and the apparent length of the tail is known to undergo most rapid changes. We shall only mention one case as an example of this phenomenon. The Comet of 1618 presented to the Danish astronomer, Longomontanus, a tail of 104° in length, while it had been measured by Kepler a few days previous, and ascertained to be only 70° long. The length of time during which a Comet is visible depends, like the apparent length of its tail, on its distance from the Earth, on its apparent position in the sky in reference to the Sun's place, and on its capacity for reflecting the solar rays of light. The time of visibility varies from a few days to more than a year, but usually does not exceed three months. The paths traversed by Comets

during the time they are visible are very variable, some being not more than 15° in six months, while others are 240°.

2. **True Dimensions of Comets.**—The distance of a Comet from the Earth is ascertained by observations made at distant localities, and found, as in the case of the Moon and Planets, by equation (14), page 73; and the distance being once ascertained, the true dimensions are found by the method already so often explained. The following Table of the true diameters of the Nuclei and Heads of several remarkable Comets is taken from Mr. Hind's valuable work on the Comets:—

TABLE XXII.—*Diameters of Nuclei and Heads of Comets.*

No.	Name of Comet.	Observer.	Diameter of Nucleus in Miles.	Diameter of Head in Miles.
1	Comet of 1815, ...	Olbers.	5300	
2	Great Comet of 1825, . .	. . . .	5100	
3	Great Comet of 1843, . .	. . . .	5000	
4	First Comet of 1780, . .	. . . .	4270	269000
5	First Comet of 1847, . .	Hind.	3500	25500
6	Fine Comet of July, 1819,	. . . .	3280	
7	Second Comet of 1811, .	Herschel.	2640	
8	Great Comet of 1807, .	Herschel.	538	
9	Great Comet of 1811, . .	Herschel.	428	1125000
10	Second Comet of 1798, .	Schröter.	125	
11	Biela's Comet, 1805, . .	Biela.	70 to 112	

It was first remarked by Kepler, and has since been confirmed by numerous observations, that the diameters of the nuclei and heads of Comets diminish as they approach the Sun; as if the gaseous matter of which they are composed were dissipated by the intense heat, and driven violently into the tail, or dispersed into space, where much of it must be lost, and never again rejoin the

Comet. The following Table contains the dimensions of Encké's Comet, and its distances from the Sun at its perihelion approach in the year 1838; and abundantly confirms the fact first noticed by Kepler:—

TABLE XXIII.—*Change in Dimensions of Encké's Comet.*

Date, 1838.	Diameter in miles.	Distance from Sun. Earth's distance = 1.
9 October,	28 100 0	1.42
25 October,	120500	1.19
6 November,	79000	1.00
13 November,	74000	0.88
16 November,	63000	0.83
20 November,	55500	0.76
23 November,	38500	0.71
24 November,	30000	0.69
12 December,	6600	0.39
14 December,	5400	0.36
16 December,	4250	0.35
17 December,	3000	0.34

The tails of Comets are frequently of great length, and formed very rapidly, sometimes at the rate of 200 millions of miles in three weeks, as in the case of the Great Comet of 1843. The following Table exhibits the greatest length of the tails of several remarkable Comets.

TABLE XXIV.—*Length of Tails of Comets.*

Name of Comet.	Length of Tail in miles.
First Comet of 1847, .	5,000000
Comet of 1744, . . .	19,000000
Comet of 1769, . . .	40,000000
Third Comet of 1618, .	50,000000
Great Comet of 1680, .	100,000000
Great Comet of 1811, .	100,000000
Second Comet of 1811, .	130,000000
Great Comet of 1843, .	200,000000

3. **Classification of Comets.**—The numerous Comets, whose orbits have been determined by astronomers, are usually arranged in five classes, according to their periodic times, and the forms of their orbits. These classes are named as follows:—

1st. Comets of Short Period.
2nd. Comets of Long Period.
3rd. Elliptic Comets beyond the limits of the Solar System.
4th. Hyperbolic Comets.
5th. Parabolic Comets.

4. **Comets of Short Period.**—The Comets of Short Period are 13 in number, and have mean distances and periodic times of revolution less than those of the planet Saturn; the excentricity of their orbits is also much less than that of any other Comets, although much greater than that of the planetary orbits. It is considered by M. Arago that only three of these Comets, viz. *Encke*, *Biela*, and *Faye*, have been proved with absolute certainty to return periodically to the Sun, and there is considerable difficulty in observing several of them in their approach to that luminary. None of these Comets are Great Comets, and some of them are only known to astronomical observers.

The following Table exhibits the principal elements of their orbits:—

TABLE XXV.—*Elliptic Comets of Short Period.*

No.	Name.	Excentricity.	Inclination.	Periodic Time in Years.	Mean Distance.	Direction of Motion.	Date of Discovery.
1	Encké.	84.78	13° 7′ 24″	3.296	2.215	Direct.	26 Nov., 1818.
2	Biela.	75.70	12 34 53	6.617	3.524	,,	27 Feb., 1826.
3	Faye.	55.50	11 22 31	7.441	3.812	,,	22 Nov., 1843.
4	De Vico.	61.76	2 55 0	5.470	3.103	,,	22 Aug., 1844.
5	Brorsen.	79.30	30 58 0	5.581	3.198	,,	26 Feb., 1846.
6	D'Arrest.	66.09	13 56 12	6.440	3.462	,,	27 June, 1851.
7	Clausen.	72.13	1 53 43	5.436	3.091	,,	February, 1743.
8	Blanpain.	68.70	9 1 16	4.810	2.849	,,	28 Nov., 1819.
9	Burckhardt.	86.40	8 1 45	5.025	2.934	,,	8 April, 1766.
10	Pons.	75.52	10 42 48	5.618	3.160	,,	21 June, 1819.
11	Lexell.	78.68	1 35 0	5.607	3.153	,,	June, 1770.
12	Pigott.	67.84	47 43 0	10.025	4.649	,,	19 Nov., 1783.
13	Peters.	75.67	31 2 14	15.990	6.320	,,	26 June, 1846.

NOTES ON THE ELLIPTIC COMETS OF SHORT PERIOD.

1. **Encke's Comet** was first seen by M. Mechain on the 17th January, 1786, near the star β Aquarii. It is a telescopic Comet, but is sometimes visible to the naked eye. It was next seen by Miss Caroline Herschel on the 7th November, 1795. It was seen for the *third* time, almost simultaneously, on the 20th October, 1805, by M. Pons at Marseilles, Professor Huth at Frankfort, and M. Bouvard at Paris. It was seen for the *fourth* time at Marseilles, by M. Pons, on the 26th November, 1818. M. Bouvard of Paris presented the calculation of its elements to the Bureau des Longitudes on the 13th January, 1819; this led to its being identified with the Comet of 1805, but its periodic time was unknown, as it was possible it might have returned several times in the thirteen years intervening. Professor Encké, who was at that time the most skilful calculator in Europe, took the subject in hands, and succeeded in showing the identity of this Comet with those of 1786 and 1795; he ascertained that its periodic time was about 1208 days, and applied himself to the task of predicting its motion, taking into account the perturbations of all the Planets. He predicted the circumstances of its return in 1822, which was invisible in Europe, but observed at Sir Thomas Brisbane's Observatory, at Paramatta, N.S.W. by Mr. Rümker. The circumstances of its motion agreed in every respect with those predicted by M. Encké, who has calculated all the subsequent appearances of this Comet, viz.: 1825, 1828, 1832, 1835, 1838, 1842, 1845, 1848, 1852, 1855.

Observations have been made on this Comet at all the periods just mentioned, and the circumstances of the motion found to agree remarkably well with the orbit assigned by Encké, with one remarkable exception, the discussion of which has led astronomers to one of the most interesting discoveries of modern science. It has been found that this Comet returns $2^h\ 30^m$ sooner than its predicted time to its perihelion position; and since it is subject to Kepler's Third Law, this indicates a shortening in its axis major, and, therefore, a gradual approach to the Sun. This curious fact led M. Encké to speculate on the existence of a resisting medium in space, which, by acting on the light structure of this small Comet, diminishes its velocity, and thus shortens its axis major, and its Periodic Time of revolution. From this hypothesis he was enabled to calculate the motion of the Comet, subject to this resistance: and it has been found that the path of the Comet, thus corrected, agrees accurately with its observed orbit. The fact thus conclusively established is one of the greatest interest, as it leads to the inevitable conclusion that all the Planets, as well as the Comets, must ultimately fall into the body of the central Sun.

Encké's Comet has not only proved the existence of an interplanetary resisting medium, but has also been the means of determining accurately the mass of the planet Mercury, near to which body it has passed frequently, and afforded, by the deflection it underwent, the means of measuring the mass of Mercury, which is proportional to the deflecting force.

2. **Biela's Comet** was first seen on the 8th of March, 1772, by M. Montaigne of Limoges; and was rediscovered by M. Pons on the 10th of November, 1805. It was again seen by M. Biela, of Josephstadt in Bohemia, on the 27th of February, 1826. It was recognised independently by M. Gambart of Marseilles, on the 9th of March. Clausen, Biela, and Gambart, all determined its elliptic elements, and found a periodic time differing little from $6\frac{3}{4}$ years.

This Comet caused considerable alarm in 1832 from the circumstance that it was certain that its nebulosity would inclose a portion of the Earth's orbit; the Earth, however, was not in any danger, as she was not near this portion of her orbit at the time of the passage of the Comet. Biela's Comet passed close to the Earth's orbit just before midnight of the 29th of October, and the Earth did not reach that point of her orbit until the morning of the 30th of November, so that there was no sufficient cause for the general alarm.

A very remarkable phenomenon attended the perihelion passage of this Comet in the latter end of 1845. It became divided into two Comets, which did not again reunite, but travelled along together in similar orbits. This unique phenomenon was noticed for the first time in America on the 29th of December. The greatest distance observed between these two fragments of Biela's Comet, before their final disappearance, was about *two-thirds* of the Moon's distance from the Earth.

3. **Faye's Comet.**—On the 22nd of November, 1843, M. Faye discovered this telescopic Comet, and afterwards calculated its elliptic elements. These were also calculated by Argelander, Goldschmidt, and Henderson; and the accuracy of the elements finally proved at the time of the reappearance of the Comet in 1851. Faye's Comet was at first thought to be the same as Lexell's Comet, generally known as the "*Lost Comet*;" but it was satisfactorily shown by Leverrier that it could not possibly be the same.

4, 5, 6. **The Comets of De Vico, Brorsen, and D'Arrest** are not considered by Astronomers as entitled to be placed in the same list as the three Comets already described. They have not yet satisfied the only sure test of Periodicity, viz., the agreement of Calculation and Observation at a predicted perihelion passage. It must be remembered, however, that it frequently happens that a Comet may be invisible during the whole time it is in the neighbourhood of the Sun. None of the *ten* remaining comets, with the exception, perhaps, of Lexell's or the *Lost Comet*, is entitled to be ranked as yet with the comets of Encké, Biela, and Faye.

7, 8. **Comets of Clausen and Blanpain.**—Many astronomers regard these Comets as the same body imperfectly observed.

9, 10. **Comets of Burckhardt and Pons.**—These Comets are also regarded by some astronomers as identical.

11. **Lexell's Comet.**—This famous Comet was discovered in June, 1770, by Messier of Paris. All attempts to represent its motion by means of a parabolic orbit having failed, M. Lexell, of the Academy of Sciences of St. Petersburg, succeeded in representing the path of the Comet by an ellipse, which corresponded to a Periodic Time of revolution of 5 years and 7 months. These results were not obtained until 1778, or two years after the time at which the Comet should have returned. This Comet passed nearer to the Earth than any other known Comet, being at 5 P. M. of the 1st of July only 363 terrestrial radii distant, or 1,400000 miles. At that time the nebulosity surrounding the nucleus subtended an angle of 2° 23′, from which it appears that its real diameter was 59000 miles. Lexell's Comet was not seen in 1776, probably on account of its proximity to the Sun at the time it was likely to be seen by European observers, rendering it invisible, unless looked for by telescopes purposely. But this Comet was never seen before or since 1770, and as it was a large Comet, this fact appeared difficult to account for. It was shown by Burckhardt, and the general result confirmed by Leverrier, that Lexell's Comet passed close to Jupiter on two occasions, at the end of May, 1767, and on the 23rd of August, 1779. On the first of these occasions Jupiter's attraction was *three* times greater on the Comet than that of the Sun; and on the second occasion, it was 225 times greater. It was owing to the first of these approaches to Jupiter that the Comet of Lexell was compelled to describe the elliptic orbit assigned by him, and recorded

in our Table; and it was owing to the second approach of the Comet to Jupiter that the form of this elliptic orbit was changed into another, of such dimensions and position, that Lexell's Comet, revolving in it, has remained invisible to us since 1770, and will probably continue to be invisible, until another planetary attraction shall again bring it within the range of our telescopes. Lexell's, or the *Lost Comet*, is one of the most interesting bodies that circulate round the Sun.

5. **Comets of Long Period.**—There are six Comets, whose Periodic time is about equal to that of Uranus; and whose orbits are nearly altogether within the orbit of Neptune. The following Table contains the particulars of their orbits, so far as they have been yet determined :—

TABLE XXVI.—*Elliptic Comets within the Orbit of Neptune.*

No.	Name.	Excentricity.	Inclination.	Periodic Time in Years.	Mean Distance.	Direction of Motion.	Date of Discovery.
1	Halley, .	96.74	17° 47′	76.78	17.9763	Retrograde	Christmas Day, 1758
2	Pons, . .	95.45	73 57	70.68	17..955	Direct, . .	July, 1812.
3	Olbers, .	93.12	44 30	74.05	17.6338	,,	Mar.6, 1815.
4	De Vico,.	95.44	84 57	73.25	14.5386	,,	Feb.20, 1846
5	Brorsen,.	97.26	19 8	74.97	17.7795	,,	July, 1847.
6	Westphal,	92.48	40 58	67.77	16.6200	,,	June, 1852.

The group of Comets described in the foregoing Table have nearly the same mean distance, and nearly the same Periodic time; and some astronomers have considered it likely that four of them may have originally constituted a single Comet. With the exception of the first and third, Halley and Olbers, their orbits cannot be considered as better than rough approximations; and it is premature to speculate on their cosmical origin.

1. **Halley's Comet.**—A remarkable Comet having shown itself in 1682, Halley, by the aid of a method suggested by Newton, determined the parabolic elements of its orbit from the observations of La Hire, Picard, Hevelius, and Flamstead. The results of this calculation are given under the date, 1682, in the following Table.

TABLE XXVII.—*Orbit of Halley's Comet.*

No.	Date.	Calculator.	Inclination.	Longitude of Node.	Longitude of Perihelion.	Perihelion Distance.	Direction of Motion.
1	1378	Langier, . . .	17° 56′	47° 17′	299° 31′	0.58	Retrograde
2	1456	Pingré, . . .	17 56	48 30	301 0	0.58	,,
3	1531	Halley, . . .	17 56	49 25	301 39	0.57	,,
4	1607	Halley, . . .	17 2	50 21	302 16	0.58	,,
5	1682	Halley, . . .	17 42	50 48	301 36	0.58	,,
6	1759	Clairaut, . . .	17 37	53 50	303 10	0.58	,,
7	1835	Damoiseau, . Pontecoulant, Rosenberger, Lehman, .	17 44	55 50	304 32	0.58	,,

Having determined the parabolic elements of the orbit in 1682, Halley calculated the elements of a Comet seen in 1607 from the observations of Kepler and Longomontanus; and also the elements of a Comet observed in 1531 by Apian at Ingoldstadt. The results of his calculations are given in the preceding Table.

It became evident to Halley that these three Comets were identical; and he ventured to predict its return at the close of 1758, or probably 1759. As this was the first prediction of the return of a Comet, it was looked for with much interest by astronomers; and as it was certain to be retarded by the attractions of Jupiter and Saturn, it became a matter of consequence to calculate the amount of retardation likely to be produced by these bodies. The task was one of very great labour, and was undertaken by Clairaut, in conjunction with Lalande and Madame Lepaute. For six months they computed incessantly, from morning to night, with but little intermission,—as Lalande states, even at meals. On the 15th of November, 1758, Clairaut announced the results to the French Academy as follows:—the Comet would be retarded 518 days by the attraction of Jupiter, and 100 by that of Saturn, causing together a delay of 618 days in the time of its perihelion passage. In announcing this conclusion, Clairaut stated the time of perihelion pas-

sage as the 13th of April, 1759; and added that, owing to the unavoidable neglect of certain small quantities in the calculation, this period might be 30 days in error, on both sides. Its perihelion passage took place on the 12th of March, just one month before the time predicted by Clairaut; in justice to whose admirable accuracy it should be added, that Laplace has shown that if Clairaut had used in his calculation the mass of Saturn, as known to modern astronomers, his prediction, instead of being 31 days in error, would have been only 13 days wrong.

Previous to the return of the Comet of Halley in 1835, many astronomers engaged in the laborious work of calculating the time of its perihelion passage. The most remarkable of these predictions were as follow:—

Damoiseau's Perihelion passage,			8 P. M.,	4th of November,	1835.
Pontecoulant's,	,,	,,	5 A. M.,	13th of November,	,,
Rosenberger's,	,,	,,	Noon,	11th of November,	,,
Lehman's,	,,	,,		26th of November,	,,

The Comet actually passed its perihelion on the 16th of November, a period well within the limits of the predictions. The differences in the predicted periods of perihelion passage arise from the omission of the attraction of the smaller planets by some of the calculators.

Rosenberger's calculation is regarded as the most elaborate; it assigns to the Earth's attraction a hastening of the perihelion passage of 15⅞ days, to Venus about 5½ days, and to Mars and Mercury together nearly 1 day. It is remarkable that the motion of Halley's Comet is opposite to that of the other five Comets, which it resembles so closely in other respects. It is the only periodic Comet endowed with a retrograde motion.

6. **Elliptic Comets beyond the limits of the Solar System.**—When a Comet makes its appearance, it is usual to calculate the elements of its orbit, on the supposition that it is parabolic, and if the parabolic elements resemble those of previous Comets, the question of the identity of the bodies is raised and discussed. But as many ellipses and hyperbolas resemble parabolas, when only a small arc near the vertex is considered, it is evident that many of our so-called parabolic Comets may turn out to be of elliptic or hyperbolic orbits. Suspicion has already alighted on twenty-two of those bodies, which are supposed, on different degrees of evidence, to have elliptic

orbits, whose aphelia are situated far beyond the boundaries of our Solar System, but still within the sphere of the Sun's attraction, by virtue of which they may one day return and revisit our lower world of planets. The following Table contains the most probable elliptic elements of the orbits of these twenty-two Comets, very few of which are sufficiently well known to us to require special mention.

TABLE XXVIII.—*Elliptic Comets beyond the Orbit of Neptune.*

No.	Name.	Excentricity.	Inclination.	Periodic Time, in Years.	Mean Distance.	Direction of Motion.	Discovery, Date of.
1	Comet of 1680,	99.99	60° 40′	8813	427.6	Direct,	1680
2	. . . .	98.32	83 47	190	33.0	Retrograde	1683
3	. . . .	99.54	72 34	3205	217.4	Direct,	1763
4	. . . .	99.92	40 45	2089	163.4	,,	1769
5	Comet of 1780,	99.99	54 23	75838	1787.9	Retrograde	1780
6	D'Arrest, .	97.34	51 31	421	56.2	Direct,	1793
7	. . . .	99.55	63 10	1725	143.8	,,	1807
8	. . . .	99.51	73 2	3065	211.0	Retrograde	1811
9	. . . .	98.27	31 17	875	91.5	Direct,	1811
10	. . . .	99.63	52 39	5444	309.5	Retrograde	1822
11	. . . .	93 54	33 32	4386	267.9	,,	1825
12	. . . .	99.93	54 4	2611	189.6	,,	1827
13	Comet of 1830,	99.94	21 16	60200	1536.0	Direct,	1830
14	. . . .	99 79	59 13	13864	577.1	Retrograde	1840
15	Bremiker, .	96.99	57 57	344	49.1	Direct,	1840
16	Comet of 1843,	99.99	35 41	376	56.0	Retrograde	1843
17	Comet of 1844,	99.96	48 36	100000	2138.0	,,	1844
18	. . . .	98.99	48 41	251	39.7	,,	1845
19	. . . .	99.24	47 26	2719	194.8	Direct,	1846
20	. . . .	98.84	29 18	401	54.4	Retrograde	1846
21	. . . .	99.78	66 59	8375	164.2	Direct,	1849
22	Expected, .	99.99	30 12	308.1		,,	1264 1556 18—

The expected Great Comet.—The attention of astronomers is very generally directed at present to No. 22 of the foregoing list, which is expected to return some time before 1860. The history of this Comet is as follows. It was first suspected by Mr. Dunthorne that the great Comet of 1264, whose tail extended over 100° of arc, was identical with a great Comet of 1556, observed by Fabricius, and whose orbit was calculated by the celebrated Halley. Additional evidence bearing on the question of identity has since come to light; and one astronomer, M. Bomme, of Middelburg, in the Netherlands, has calculated the time of its next return, allowing for the attractions of Jupiter, Saturn, Uranus, and Neptune, and partially of the Earth, Venus, and Mars. The elements of the orbit of 1556 are uncertain; but M. Bomme has made his calculations for two orbits, one given originally by Halley, and the other recently by Mr. Hind. He has found that on the first assumed orbit, the Comet is due at its perihelion on the 22nd of August, 1860, and on the assumption that Mr. Hind's orbit is correct, the time of its perihelion Passage would occur about two years earlier, viz., on the 2nd of August, 1858. It is confidently hoped by many that the next three years will solve this very interesting problem. It is nearly certain that it will be solved either affirmatively or negatively; for it is very unlikely that a Comet of such size and brilliancy as that of 1264 and 1556 should escape our notice, if it really returns. And yet we know so little of the changes undergone by Comets, and have witnessed such remarkable alterations in them, that we cannot even be certain that a Comet once great and visible to the naked eye will always continue so.

7. **Hyperbolic Comets.**—We have seen that there are forty-one Comets whose orbits are better represented by an ellipse than by a parabola. In like manner there are seven Comets known, whose orbits are better represented by an hyperbola than by a parabola. These Comets having once passed the Sun at their perihelion, can never again return, and must move off into space, until they come within the sphere of attraction of some other star, under whose influence they commence to describe another orbit, the nature of which depends upon several circumstances well known to physical astronomers, but which cannot be explained in an elementary work. As soon as the wandering Comet describes an elliptic orbit, it remains a prisoner to the star to whose system it has arrived, and becomes in every respect analogous to the forty-one Comets which we have already mentioned. The following Table con-

tains the scanty information we possess respecting these singular bodies.

TABLE XXIX.—*Orbits of the Hyperbolic Comets.*

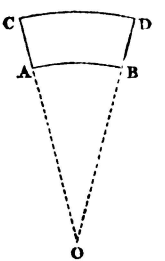
No.	Excentricity.	Inclination.	Perihelion Distance.	Direction of Motion.	Date.
1	100.50	77° 5′	4.0435	Direct,	June, 1729.
2	100.91	11 15	0.9035	,,	April, 1771.
3	102.83	83 20	1.4329	,,	Aug., 1774.
4	101.16	63 0	0.8550	Retrograde,	Dec., 1818.
5	100.17	54 37	1.0500	Direct,	Sept., 1824.
6	100.02	53 5	0.6184	,,	Jan., 1840.
7	100.02	52 44	1.6163	,,	May, 1843.

8. **Parabolic Comets.**—The orbits of 281 Comets have been calculated, according to Mr. Hind's Catalogue, from 370 B. C., to 1852 A. D., inclusive. Of these Comets, forty-one probably move in elliptic, and seven in hyperbolic, orbits; and of the remaining 233 Comets, many are doubtless reckoned more than once. Although said to move in parabolic orbits, it is not to be supposed that any body could move in a parabola or circle for any continued space of time, as the disturbing attractions of the planets would speedily compel it to change the form of its orbit. It is, therefore, certain that the two hundred and odd Comets, called parabolic, are really moving in elliptic or hyperbolic orbits; and, consequently, either belong to the Solar System permanently, or are only transient visitors. The observation of Comets and calculation of their orbits is at present one of the most attractive fields of research for astronomers; and it cannot be doubted that the labours of the last fifty years in this branch of Astronomy have laid up for our successors an accumulation of facts which will render it comparatively easy for them to discover the true character of many Comets, should they again visit the neighbourhood of the Sun.

APPENDIX.

ON THE USE OF THE GLOBES.

MANY questions in Astronomy which require, for their complete solution, the use of spherical Trigonometry, may be determined with sufficient accuracy for practical purposes with the aid of the terrestrial and celestial globes. As no difficulty will be found in applying the globes to the solution of the following problems, we shall dispense with a verbal description of their nature and construction; and merely instruct the reader, as a preliminary, how to make a simple apparatus, which will greatly assist him in their use.



On a piece of card take any point O as centre, and, for a twelve-inch globe, describe an arc AB of convenient length, with a radius of six inches; with another radius OC describe an arc so that AC shall be one inch and a half or two inches high. Draw the radii OAC and OBD, and with a sharp knife cut out the space ABDC. For an eighteen-inch globe the radius OA should be nine inches. The arc AB will exactly fit the curvature of the globe, and the edges AC and BD will be perpendicular to its surface. When this card, which may be called the *Index Card*, is applied to the brass meridian, with either extremity over a place or star, it enables us to read off the latitude or declination with great precision to a quarter of a degree. The same mode of using it enables us to read off amplitudes and azimuths on the horizon.

PROBLEMS WHICH MAY BE SOLVED BY THE TERRESTRIAL GLOBE.

Problem 1.

To find the Latitude and Longitude of a given Place.

1. Bring the place to the brass meridian.
2. Apply the index card with either extremity over the place.
3. Read off the longitude E. or W. on the equator, and the latitude on the brass meridian where it is cut by the card.

Exercises.—Find the latitude and longitude of Paris, Vienna, Philadelphia, and Cape Horn.

Problem 2.

Given the Latitude and Longitude, to find the Place.

1. Bring the degree of longitude on the equator under the brass meridian.
2. Apply the edge of the index card at the given latitude. The place under its extremity is the required place.

Problem 3.

To find the Distance from one Place to another on the Earth's Surface.

1. On a slip of paper about a quarter of an inch in breadth, measure the distance of the two places.
2. Apply it to the equator, and multiply the corresponding number of degrees by 60, to obtain the distance in geographical miles; by 69.1, in statute miles.

Exercises.—Find the distance in statute miles between New York and Galway.—In geographical miles between Rio de Janeiro and the Land's End.—In statute miles between Balaklava and Sinope.

Problem 4.

To find the length of a Degree in Longitude corresponding to a given Latitude.

1. At the given latitude, measure on a slip of paper the distance between two meridians which differ in longitude by 15 degrees.

2. Apply the paper to the equator, and multiply the corresponding number of degrees by 4, to obtain the number of geographical miles; by 4.6, to obtain the number of statute miles in a degree of longitude.

Exercise.—Find the length of a degree of longitude at the following places:—London, Vienna, Sebastopol, Cape of Good Hope.

Problem 5.

To find the Relative Amount of Land and Water on the Surface of the Earth.

Having procured the segments of paper which are used for covering a terrestrial globe, cut away with a pair of scissors the part which represents land; weigh this in a balance, then weigh that which represents the water: the ratio of the weights thus found will be the required ratio; in this way the proportion of land and water has been found to be 1 : 2.815. By a similar process may be found the area of any portion of the Earth's surface, such, for instance, as the drainage areas of the great rivers, &c.

Problem 6.

To find the Antipodes of a given Place.

1. Set the poles of the Earth in the horizon.
2. Bring the place to the eastern horizon, and with the index card read the number of degrees north or south of the east point.
3. Set the edge of the card on the western horizon at the same number of degrees south or north of the west point, and its extremity will indicate the Antipodes.

Exercises.—Find the Antipodes of the following places:—London, Berlin, Constantinople, Lima.

Problem 7.

Given the Hour of the Day at a Place, to find the Hour at another Place.

1. Find the longitude of each place.
2. Convert the difference into time, at the rate of 15° for an hour; if the place be west, subtract from, if east, add to, the given time.

The same may be done by means of the hour circle:—

1. Bring the given place to the meridian.
2. Set the hour circle to the given time.
3. Turn the globe till the other place comes to the meridian; the hour circle will show the required time.

Exercise.—When it is 10 A. M. in Dublin, what is the hour in New York?—If it be 4 P. M. in London, what o'clock is it at Sebastopol?—If it be 7½ P. M. in Edinburgh, required the time in Calcutta.

Scholium.—If the local time and that at Greenwich be known, we can find the longitude E. or W. by this problem.

Problem 8.

To find the Sun's Longitude, Right Ascension, and Declination, the Day being given.

1. Find the day of the month on the ecliptic; and at this point read the Sun's longitude.
2. The declination and right ascension of this point are found in the same manner as the latitude and longitude of a place in Problem 1.

Exercises.—Required the longitude, declination, and right ascension of the Sun, on the following days:—May 16, July 27, October 10, December 11.

The declination of the Sun may also be readily found by means of the *Analemma:* at the day of the month, stretch the straight edge of a slip of paper across the analemma parallel to the equator; the degree cut off on the medial line is the declination.

Note.—The declinations, right ascensions, latitudes, and longitudes of all the heavenly bodies, at the Greenwich noon of each day, may be found with much greater accuracy in the Nautical Almanac, which, by a recent order of the Lords of the Admiralty, may be purchased for half-a-crown.

Problem 9.

To find the Time of the Sun's rising and setting at a given Place, on a given Day; and also the lengths of the Day and of the Night.

1. Elevate the pole to the latitude of the place.
2. Bring the Sun's place in the ecliptic to the meridian, and set the hour circle at XII.
3. Turn the globe till the Sun's place is brought down to the eastern horizon; the hour circle will tell the time of rising.
4. Turn the globe until the Sun's place comes to the western horizon; the hour circle will tell the time of setting.

Twice the time of setting gives the length of the day. Twice that of rising, the length of the night.

Exercises.—Required the times of rising and setting of the Sun, and the lengths of the days and nights at the following places and days:—London, 18th August; Edinburgh, 17th June; Dublin, 17th

March; Vienna, 4th December. Find the length of the longest day at New York, Rio de Janeiro, Hawaii, Bermuda.

Definition.—After sunset and before sunrise, we enjoy for a short time a partial light, called *Twilight.* Twilight lasts while the Sun's distance below the horizon is less than 18°.

Problem 10.

To find the Time of the beginning and end of Twilight; also the duration of Twilight at a given Place.

1. Elevate the pole to the latitude of the place, and screw the flexible quadrant over the latitude.
2. Find the Sun's place, and bring it to the meridian, and set the hour circle at XII.
3. Bring the Sun's place below the eastern horizon until it coincide with the eighteenth degree on the quadrant. The hour circle will then mark the beginning of twilight.
4. Bring the Sun's place below the western horizon, and in the same way may be found the end of twilight.
5. To find the duration of twilight, find the times of sunrise and sunset by Prob. 9. The differences between these and the times of beginning and end of twilight will be the durations of morning and evening twilight.

Exercises.—Find the beginning and end, and also the duration, of twilight, at London, on 17th August; Liverpool, on 4th November; Archangel, on 14th March.

Problem 11.

To find the Sun's Altitude and Azimuth at a given Place, the Day and Hour being known.

1. Elevate the pole to the latitude of the place, and screw the flexible quadrant over the latitude.
2. Having found the Sun's place in the ecliptic, bring it to the brass meridian, and set the hour circle to XII.
3. Turn the globe till the hour circle marks the given hour.
4. Bring the graduated edge of the flexible quadrant over the Sun's place: the number of degrees between it and the horizon will be the altitude; and the number of degrees intercepted on the horizon from the north point will be the azimuth.

Exercises.—Find the altitude and azimuth of the Sun at London, on the 18th October, at 3½ P. M.; at Paris, on the 3rd March, at 9½ A. M.; at Ceylon, on the 5th January, at 7 A. M.

Problem 12.

At a given Place, the Day and Hour being given, to find how the various Regions of the Earth are situated with regard to Day, Night, Twilight, &c.

1. Elevate the pole according to the declination of the Sun.
2. Bring the place to the meridian, and set the hour circle to the given time.
3. Turn the globe until the hour circle marks XII.

Then all regions above the horizon enjoy daylight; all below, night.

It is noon or midnight to every place under the meridian, according as it is situated above or below the horizon.

That place under the meridian whose latitude is the same as the Sun's declination has the Sun vertical.

At all places along the western horizon the Sun is rising; at all places along the eastern horizon he is setting.

At all places situated 18° below the western horizon the day is breaking, i. e. twilight is commencing; at all places 18° below the eastern horizon night is setting in, i. e. twilight is ending.

If the globe be turned on its axis, all those places round the elevated pole which do not sink below the horizon, i. e. whose colatitude is less than the Sun's declination, continually enjoy full daylight; while all the corresponding places round the depressed pole are in darkness.

If the place does not sink below or rise above the horizon more than 18°, in the former case twilight lasts all night, in the latter case all day.

Note.—The circumstances discussed in this Problem may be clearly conceived by placing a ball, which we shall imagine to be the Sun, at a considerable height, exactly over the globe, and turning the globe on its axis from west to east.

Exercise.—Discuss the circumstances required in this Problem, with regard to Liverpool, on the 14th July, at 4 P. M.

Problem 13.

To find that Portion of the Year during which the Sun never sets, at a given Place within the North Frigid Zone.

1. At the degree on the brass meridian north of the equator, which is equal to the colatitude of the place, set the index card.
2. Make the globe revolve, and mark the days on the ecliptic which pass under the edge of the card.

The day before the 21st June will be the beginning, and the other the end, of that portion of the year during which the Sun never sets.

Corollary.—The portion of the year during which the Sun never rises may be found in precisely a similar manner, reading south for north, and 21st December for 21st June.

Exercises.—Find the season of continual day at the south part of Nova Zembla.—Perpetual night at Barrow Straits.

Problem 14.

To find the Portions of the Year during which Twilight lasts from Sunset to Sunrise, at a Place within the North Frigid Zone.

1. At the degree of the brass meridian north of the equator, which is equal to the colatitude diminished by 18°, set the index card.
2. Make the globe revolve, and mark the days on the ecliptic which pass under the edge of the card.

In the intervals between these days, and those determined in the last Problem, twilight lasts from sunset to the following sunrise.

Exercises.—Determine the seasons in which there is twilight all night at Reikiavik.—At the north Cape.

Scholium.—In Problem 13 it is evident that the condition is, that the Sun's declination shall be greater than the colatitude; in Problem 14, that the Sun's declination shall be greater than the colatitude, minus 18°.

Scholium.—The lowest latitude at which twilight can last all night is 48½°.

Problem 15.

Given the Place, the Day, and Hour, to find that other Place in which at the same instant the Sun is Vertical.

1. Find the Sun's declination (Prob. 8).
2. Bring the place to the meridian, and set the hour circle to the given time.
3. Turn the globe towards the west if the time be A. M., towards the east if P.M., until XII. come on the meridian.
4. Set the index card at that degree which marks the declination of the Sun; at the place which is under the edge of the card the Sun is vertical.

Exercises.—When it is 4 P. M. in London on the 18th August, where is the Sun vertical?—When it is $3\frac{3}{4}$ P. M. in Liverpool on the 16th August, where is the Sun vertical?

Problem 16.

To find the Seasons of Day, Night, and Twilight, at the North Pole.

If the north pole be elevated to 90°, the equator will coincide with the horizon; and in this position the northern half of the ecliptic, from the first point of Aries to that of Libra, will be above the horizon; the Sun, therefore, will be always above the horizon from 21st March to 23rd September, i.e. for 186 days. By means of the flexible quadrant it may be ascertained that the days on which the Sun has a south declination of 18°, and, therefore, in this position of the equator, a depression below the horizon of 18°, are 13th November and 29th January. From this it is evident, that after the season of continual sunlight comes a twilight which lasts from 23rd September until 13th November, i. e. for 51 days; then a season of continual night from 13th November until 29th January, i. e. a period of 77 days; and then, again, a season of twilight from 29th January until 21st March, another period of 51 days, when the Sun once more rises above the horizon.

Scholium.—It is evident that to the inhabitant of the North Pole the Sun seems to move in a circle parallel to the horizon, and that on the 21st March and 23rd September he seems to describe the horizon itself.

Problem 17.

To find the Region of the Earth in which a Lunar Eclipse is visible, the time of its occurrence at Greenwich being given.

1. Find the place where the Sun is vertical at the beginning of the Eclipse, by Prob. 15.
2. Turn the globe round until XII., corresponding to *midnight*, comes on the meridian.
3. Elevate the opposite pole according to the declination of the Sun.

Then all places above the horizon see the beginning of the eclipse. Repeat the process for the time of the end of the eclipse, and we get another hemisphere to which the end of the eclipse is visible. To those regions which are common to both hemispheres, thus found, the whole eclipse is visible.

Exercises.—On October 25th, 1855, there was a lunar eclipse commencing at $5^h\ 40^m$ A.M., and ending at $9^h\ 15^m$ A.M., Greenwich time; determine whether the beginning and end of this eclipse were visible at Dublin.—In 1856 there occurred two eclipses of the Moon: the one on April 20th, the time of beginning being $7\frac{1}{2}$ A.M.; of ending $10\frac{1}{2}$ A.M., Greenwich time. Another on October 13th: time of beginning $9^h\ 20^m$ P.M.; of ending, half an hour after midnight. Find the regions of the earth to which these eclipses were visible.

PROBLEMS WHICH MAY BE SOLVED BY THE CELESTIAL GLOBE.

Problem 1.

To find the Declination and Right Ascension of a Heavenly Body.

Bring the body to the meridian; apply the index card, and read off its declination and right ascension.

The right ascension and declination of the Moon and of the Planets must be obtained from the Nautical Almanac.

Problem 2.

Being given the Declination and Right Ascension of a Heavenly Body, to find its place on the Globe.

1. Bring the degree of the equator which marks the right ascension to the brass meridian.
2. Apply the index card to the meridian at the given declination; the place of the body will be marked by the edge of the card.

Note.—The place of the Sun, Moon, or Planet, may be conveniently marked on the globe, by affixing to the place thus found a small fragment of white paper moistened with the tongue.

Problem 3.

To find the Latitude and Longitude of a Star.

1. Place the upper end of the flexible quadrant on the pole of the ecliptic.
2. Bring the graduated edge to the star.

The degree over the star gives the latitude; that cut off on the ecliptic, the longitude.

Definition.—The north extremity of a magnet needle, horizontally suspended in these regions of the globe, points to the *west* of the true north. The number of degrees by which it deviates from the north is called the *variation of the compass*,—it is different in amount in different places. We may assume that, for all purposes connected with the use of the celestial globe, the present amount of the variation is in—

London, 22°; Dublin, 26°; Edinburgh, 24°.

Problem 4.

How to draw a Meridian Line.

FIRST METHOD.

On a horizontal surface, set in a fixed position in the open air, strike out with a compass three or four concentric circles at convenient intervals; at the centre erect a piece of straight wire perfectly vertical; mark the place where the extremity of the shadow of the wire in the Sun passes over the circumference of the same circle before and after noon: join these points with the centre. The line which bisects the angle at the centre will be a meridian line pointing exactly north and south. The object of several circles is lest a cloud might interfere with the required observation.

SECOND METHOD.

The meridian, or north and south line, may be found by means of a mariner's compass, care being taken to allow the proper amount for the variation of the compass.

Problem 5.

To set the Celestia Globe, in a Position similar to that of the Heavens, at a given Place, at a given Day and Hour.

1. Set the globe due north and south.
2. Elevate the pole according to latitude of the place.
3. Bring the Sun's place in the ecliptic to the meridian, and set the hour circle at XII.
4. Turn the globe towards the east or west, according as the hour is in the afternoon or forenoon, for the given interval from noon.

The globe will then be in a position exactly similar to that of the heavens: so that if a pencil with its flat end be placed on the globe, over the representation of any star, or the place of a planet, its direction will point out the position of that body in the heavens.

Exercises.—Determine the stars which are rising, setting, and on the meridian, at 7½ P.M. on the 23rd October, at London.—What star

is vertical at midnight on 16th December, at Venice?—Determine the star which lies due west at an altitude of 25° above the horizon, at 8½ P. M. on the 31st of August, at Dublin.

Problem 6.

To determine the Time of Rising, Setting, and Culmination of a Heavenly Body at a given Place.

1. Elevate the pole to the latitude of the place.
2. Bring the Sun's place in the ecliptic to the meridian, and set the hour circle at XII.
3. Turning the globe from east to west, bring successively the body to the eastern horizon, the meridian, and the western horizon.

The hour circle will mark the times of rising, culmination, and setting.

Exercises.—Find when Aldebaran rises on the 10th September at Edinburgh; when *α* Ophiuci comes to the meridian on 4th August at Dublin; when Sirius sets on 8th April at Liverpool.

If the body be the Moon or a planet, its place for the *noon* of that day is taken from the Nautical Almanac, and marked on the globe. As the Moon's motion is so rapid, a considerable change in her position takes place in the interval from the preceding noon. To take this into account, proceed as in the Problem: and, having found the time, find the Moon's place for *that time* in the Nautical Almanac, and bring this place to the eastern horizon, meridian, and western horizon, and we shall obtain correctly the time of the Moon's rising, culmination, or southing, as it is sometimes called, and of her setting.

Exercises.—Find when Jupiter rises, sets, and comes to the meridian at Liverpool on the 16th of July, 1856.—Find the time of the Moon's rising, setting, and southing at Edinburgh on the 15th September, 1856.

Definitions.—A star is said to rise or set *cosmically* when it rises or sets at sunrise.

A star is said to rise or set *acronically* when it rises or sets at sunset.

A star is said to rise or set *heliacally* when the Sun is just so much depressed below the horizon as that the star may be seen; for stars of the first magnitude the depression should be 12°.

Problem 7.

The Latitude of a Place being given, to find the Day of the Year when a Star rises and sets Cosmically, when Acronically.

1. Elevate the pole to the latitude of the place.
2. Bring the star to the eastern horizon, and observe the day marked

on the ecliptic, where it is cut by the *eastern* horizon, for cosmical rising; by the *western* horizon, for acronical rising.

3. Bring the star to the western horizon, and observe the day on the ecliptic where it is cut by the *eastern* horizon for cosmical setting; by the *western*, for acronical setting.

Exercise.—At what time does Arcturus rise acronically in the latitude 37° 45′ N.?

Problem 8.

The Latitude being given, to find when a Star rises or sets Heliacally.

1. Elevate the pole to the latitude of the place.
2. Over the degree of latitude screw the flexible quadrant.
3. Bring the star to the eastern horizon.
4. Bring the degree on the flexible quadrant, which is 12° below the horizon, to coincide with the ecliptic. This will mark the day of heliacal rising.
5. Bring the star to the western horizon: and in a similar manner we obtain the day of heliacal setting.

Definition.—Long-continued observation of the heavens shows that the intersection of the ecliptic and equator moves back, i. e. from E. to W. along the ecliptic; and that in consequence of this retrograde motion of the first point of Aries to meet the Sun, the season of the equinox *precedes*, as it were, its due time. For this reason the motion is called the *Precession of the Equinox;* its amount is 52″.24 per annum, or very nearly 1° in 72 years. It is evident that if we know the longitudes of the stars for one epoch, we can calculate them for any other by increasing them by 50″.24 for each year.

Problem 9.

To prove the Precession of the Equinoxes by comparing Modern with Ancient Observations.

It may be shown (Prob. 7) that Arcturus rises acronically at the latitude 37° 45′ N. on the 1st April, or about 100 days after the winter solstice. From a passage of Hesiod, who lived at Ascra, in Bœotia, lat. 37° 45′ N., we learn that this star rose in his days at the setting of the Sun 60 days after the winter solstice. This can only be accounted for by supposing the intersection of the ecliptic and equator to have moved back upon the ecliptic, since Hesiod's time, through a number of degrees which corresponds to 40 days of the Sun's ecliptic motion,

$$365.25 : 40 :: 360° : 39°.4.$$

If 39°.4 be reduced to seconds, and divided by 2735 years, the interval, according to Sir I. Newton's chronology, between Hesiod's time and the present, the result gives for the backward motion of ♈ about 51″.

ANOTHER PROOF.

Sir Isaac Newton, in his chapter on the "Chronology of the Greeks," says, with reference to the constellations, that "they all relate to the *Argonauts* and their contemporaries, and to persons one or two generations older, and nothing later than that expedition was delineated there originally." From this, and the story commonly believed among the Ancients—that Musæus, an Argonaut himself, was the first who constructed a celestial globe—Newton infers that this sphere was constructed for the use of this celebrated expedition. In the description of this sphere, as given by Eudoxus, the *equinoctial colure*, or great circle passing through the poles and equinox, passed through the middle of the constellation of the Aries. According to Newton's calculation, this great circle in the end of the year 1689 cut the ecliptic in longitude 36° 44′; and if we assume as the time of the expedition of the Argonauts 956 B. C., this backward motion in 2645 years will give about 50″ per annum for the retrogression of ♈.

Definition.—At harvest time the Sun is in or near Libra, and consequently the Moon when full must at this time of the year be in the opposite quarter of the heavens, i. e. in or near Aries. In this position the full Moon will rise on successive evenings very nearly at the same time that the Sun sets. This circumstance, so favourable to the labours of the husbandmen at this season, has procured for the phenomenon the name of the *Harvest Moon*.

Problem 10.

To explain, with the aid of the Celestial Globe, the Phenomenon of the Harvest Moon at a given Place.

1. As the Moon describes the ecliptic in $27^d\ 7^h\ 43^m$, her daily motion is about 13°. Having procured seven small fragments of white paper, affix them to the globe at the first point of Aries, and at 13°, 26°, and 39° distance on each side; these pieces of paper will nearly represent the places of the Moon in the heavens for seven successive days.
2. Elevate the pole to the latitude of the place.
3. Bring the Sun's place, the first part of Libra, or thereabouts, to the meridian, and set the hour circle to XII.

4. Turn the globe westward, and observe the times of rising of the Moon as marked by the pieces of paper.

By comparing these times we shall perceive that on the first of the seven days the Moon will rise about an hour, on the second about 40 minutes, on the third about 20 minutes, before sunset. On the fourth day she will rise when the Sun is setting; and on the fifth, sixth, and seventh days, at intervals after sunset of 20, 40, and 60 minutes respectively.

The short interval, of about 20 minutes, between successive risings of the Moon at this season, is to be attributed to the small inclination between the ecliptic and the horizon; when, on the other hand, the Sun in spring is in Aries, and the Moon when full in Libra, the inclination of the ecliptic is a maximum. Under these circumstances, a similar mode of using the globe will show that the interval between successive risings of the Moon is about $1\frac{1}{2}$ hours between the first and second, and between the sixth and seventh days and about $1\frac{1}{4}$ hours for the remaining days; and that on the first and seventh days there is an interval of about 4 hours between the Moon's rising and sunset; on the second and sixth an interval of about $2\frac{1}{2}$ hours; and on the third and fifth about $1\frac{1}{2}$ hours.

MISCELLANEOUS QUESTIONS IN ASTRONOMY AND THE USE OF THE GLOBES.

1. What is the length of the day at Paris on 16th October?

2. Find the duration of twilight on the 21st November at Liverpool.

3. At how many degrees E. of N. does the Sun rise on the 23rd April at New York?

4. Where is the Sun vertical when it is $2\frac{1}{2}$ P. M. at London on the 20th February?

5. At what hours of the day is the Sun due east at Liverpool on the 28th August?

6. Find the Sun's altitude and azimuth on midsummer day at Edinburgh at 5 A. M.

7. Find the number of statute miles in a degree of longitude at Calcutta.

8. Find the distance in statute miles between Melbourne and Cape Horn.

9. What is the length in statute miles of the chain of the Andes from Guayaquil to the 50th parallel of south latitude?

10. When does the Sun rise at Bermuda on midwinter day?

11. How long does Alpherat remain above the horizon on the 26th and 27th July at Dublin?

12. It is $3\frac{1}{2}$ P.M. at Philadelphia on the 6th November, 1856,—at what place is the Moon vertical?

13. What stars are constantly above the horizon at Bristol?

14. During what part of the year does twilight last all night at London?

15. What is the greatest and least altitude of the Sun at Vienna?

16. Captain Cook was killed at a place whose latitude was 20° N., and longitude 155° W.; name the place.

17. What celebrated volcano is situated at latitude 38° N., and longitude 15°.45′ E.?

18. Find the declination and right ascension of Arcturus in the constellation Boötes?

19. In what quarter of the heavens will α Lyræ appear at 9 o'clock on the evening of the 14th September at Dublin?

20. On the 18th September, 1855, the right ascension of Venus was 13^h; was she a morning or an evening star?

21. On the 16th November, 1856, the position of Jupiter is very nearly coincident with the first point of Aries; at what hour does he rise at Liverpool?

22. On the 27th September, 1856, the planet Jupiter is in opposition to the Sun; what is his longitude?

23. What are the seasons of day, night, and twilight, at the South Pole?

24. At what instant of the year is a man unable to see his shadow at Madras?

25. At 10½ P.M. at Edinburgh, on 26th August, what star is rising 60° E of N.?

26. At 10 o'clock in the evening, at Dublin, I observed the pointers of the Great Bear on the meridian and above the Pole; what was the day of the year?

27. At what hour of the night does Spica Virginis rise at Glasgow, on the 2nd April?

28. If it be 20 minutes past 5 o'clock in the evening at Liverpool, what is the hour in Quebec?

29. If Pollux be rising when the pointers of the Great Bear are on the meridian below the Pole, what is the latitude?

30. The Moon is in her third quarter at midnight on the 26th May 1856; assign her position on the celestial globe.

31. Find the antipodes of Vienna.

32. Determine the day of the year in which γ Draconis is in the zenith of London at 10 P. M.

33. What is the meridian altitude of the Sun at Madrid, on the 5th June?

34. If the meridian altitude of the Sun at Liverpool be 42°, what is the month and day of the month?

35. When does Rigel, in the constellation Orion, rise heliacally at Paris?

36. At what period of the year does the star β, in the constellation Cassiopea's Chair, appear exactly north-east in the heavens, at 11 P. M. at Dublin?

37. A person at Bombay observes that Arcturus rises at 8 P. M.; what is the day of the year?

38. When does the Moon rise at Dublin on the 14th of May, 1856?

39. When does she set at the same place, and on the same day?

40. By how many statute miles is the distance between Galway and New York, measured on the surface of the Earth, greater than the right line joining them?

41. The projector of a submarine telegraph, in his prospectus, represents the distance from St. John's, Newfoundland, to Galway, as 1647 nautical miles, and from St. John's to Liverpool as 2020 nautical miles; is this true?

42. If Capella be in the meridian when Deneb is rising, what is the latitude of the place?

43. At what hour on the night of the 10th of May, at Liverpool, will β and γ Draconis lie on the same vertical, and what is their common azimuth?

44. At what hour does α Aquila pass the meridian of Dublin on the night of the 11th June?

45. In what degree of north latitude is β Ursæ Majoris vertical when crossing the meridian?

46. Required the difference of duration between the longest day at Algiers and St. Petersburgh.

47. If a navigator sail in a westerly direction, and at such a rate as would enable him to circumnavigate the globe in 600 days, by how much will his day, or interval between successive noons, exceed 24 hours?

48. If he sail east, by how much will his day fall short of 24 hours?

49. If two navigators, one sailing west and the other east, from the same meridian, encounter each other on the opposite meridian, by how much will their reckoning of time differ?

50. The longitude of Sirius on the 1st of January, 1856, was 99°; what was its longitude at the commencement of the Christian Era, allowing 50".24 for the mean amount of precession.